Week 04: Magnetic Actuation

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Lecture Overview

- Magnetic Force and Torque
- Pulling objects with Field Gradients
- Rolling and rocking motion
- Induced Forces and Resonant Actuators

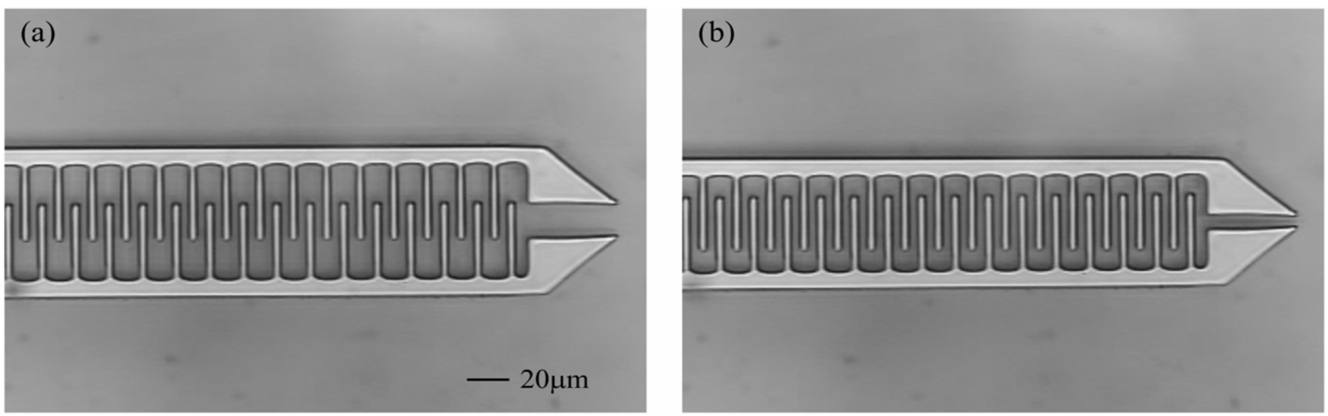
- Next week: Swimming at Low Reynolds Number

When charges do not move

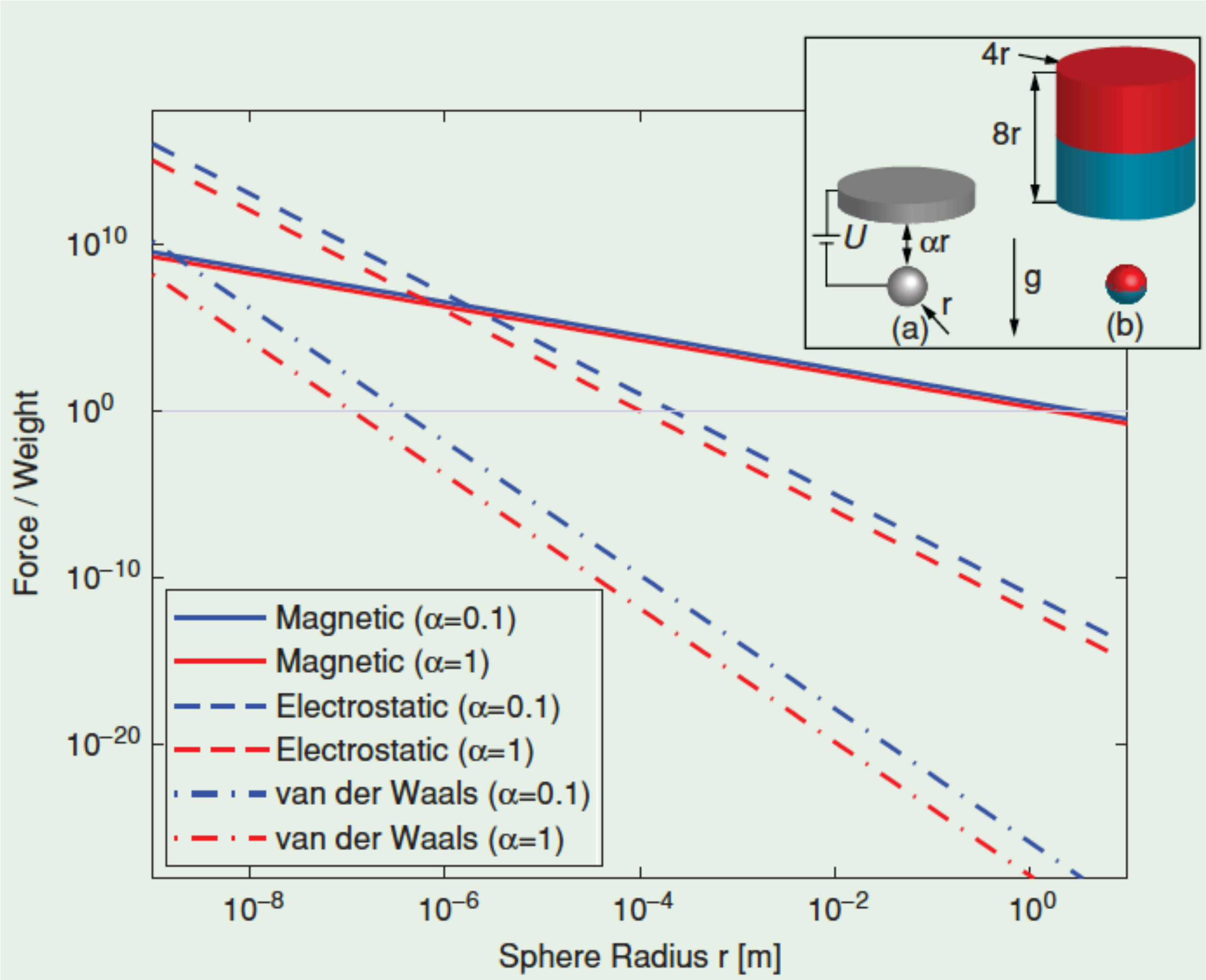
- Electrostatic force between electrodes: Coulomb's Law
- Deformation of the medium (dielectric elastomer)
- Alignment of permanent dipoles and generation of stress (piezo)
- Energy dissipation: Joule heating (shape memory)
- Van der Waals Interactions
  - Between atoms
  - Result of electron charge distribution
  - Interaction potential

Electrostatics

- Relatively long range interactions at microscale
- Induced by ionization or polarization
- Major function and failure mechanism of MEMS devices



Scaling of Attractive Forces



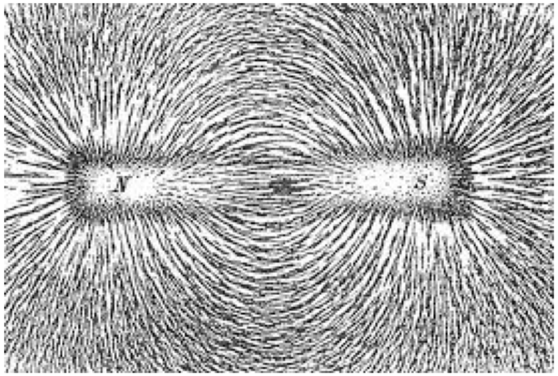
When charges move

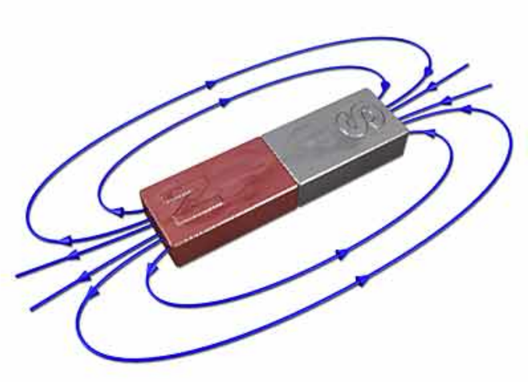
- **Maxwell's Equations**

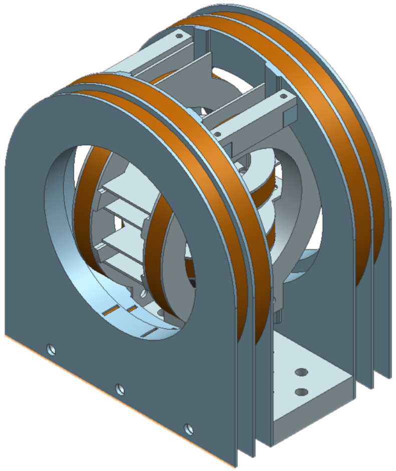
Ampère	$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$	Moving charge (J) produces magnetic field
Faraday	$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$	Change in flux (B) produces electric field
Flux	$$\nabla \cdot \mathbf{B} = 0$$	No magnetic monopoles exist
Gauss	$$\nabla \cdot \mathbf{D} = \rho$$	Electric flux is produced by electric charges

Magnetism

- Moving electrical charges generate a magnetic field.
- When a field is generated in a volume of space, there is a change in energy of that volume
- The manifestation of a magnetic field
  - acceleration of an electric charge moving in the field
  - force on a current-carrying conductor (electric motor)
  - force and torque on a magnetic dipole (e.g. a bar magnet)







Magnetic Materials

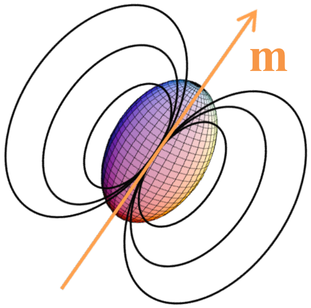
- In most atoms, electrons occur in pairs and spin in opposite directions. When electrons are paired together, their opposite spins cause their magnetic fields to cancel each other and no net magnetic field exists

- In materials with unpaired electrons these can align with an external field and exhibit a net magnetic field

- Most materials can be classified into
  - **Diamagnetic**: no unpaired electrons, small attenuation of external field
  - **Paramagnetic**: small number of unpaired electrons, small intensification
  - **Ferromagnetic**: large number of unpaired electrons, large intensification, some can retain magnetic field (permanent magnet)

Magnetic Dipole

- Magnetic flux is divergence free $\nabla \cdot \mathbf{B} = 0$
- A magnetic pole (similar to electric charge) on one end and a second, equal but opposite, pole on the other
- Generates magnetic field, with field lines going from the north to the south pole
- Described by a vector **m** called magnetic moment
  - **m** points from south to north pole
  - The magnitude of **m** is called the dipole strength (in A·m^2)

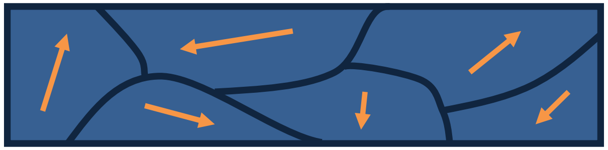


- Macroscopic magnetic properties are a consequence of magnetic moments associated with individual electrons

Magnetic Materials

- **Paramagnetism**
  - The dipoles of unpaired electrons tend to align in parallel to an external field, but lose alignment once the external field is removed

- **Ferromagnetism (analogous to ferroelectrism)**
  - In ferromagnetic materials the dipoles tend to align spontaneously without any applied field. Two nearby dipoles will tend to align in the same direction to reduce their exchange energy
  - At long distances (many thousands of atoms), in a non-magnetized ferromagnetic material, the dipoles in the whole material are not aligned.
  - Rather, the dipoles are organized into magnetic domains and are aligned at short range, but at long range adjacent domains are anti-aligned (there is little or no net field produced by such a material).



Magnetic Field and Flux Density

- What is the difference between **H** and **B**?
  - A magnetic field **H** (in A·m^{-1}) gives rise to a magnetic induction or flux density **B** (in T) in a material with permeability μ (in N·A^{-2} or henry·m^{-1} or T·m·A^{-1})

$$\mathbf{B} = \mu \mathbf{H}$$

- In general, as **B** and **H** are vectors, μ is a 3x3 tensor and the relationship between **B** and **H** is nonlinear and anisotropic, even temperature dependent

- **H** does not depend on the medium, whereas **B** does.

- **H** is defined as a modification of **B** due to magnetic fields produced by material media.

Magnetic Permeability

$$\mathbf{B} = \mu \mathbf{H}$$

- B is a measure of how a material reacts to a magnetic field H, and we can use the permeability μ to classify magnetic materials
  - Linear and isotropic materials (μ is a positive scalar)

$$\begin{aligned} \mu &= \mu_0 \mu_r \\ \mu_0 &= 4\pi \times 10^{-7} \text{ Tm/A} \end{aligned}$$

μr is the relative permeability

- diamagnetic $\mu_r < 1$ — decrease in flux density, repelled by magnet
- paramagnetic $\mu_r = 1...10$
- Ferromagnetic $\mu_r \gg 10$ — Concentrate flux, attracted by magnet
- Vacuum/air $\mu_r = 1$

Ferromagnetic: Commonly known as *magnetic materials*. Examples are iron, nickel, cobalt and their alloys

Magnetic Permeability

$$\mathbf{B} = \mu \mathbf{H}$$

- For example, inside iron (and other ferromagnetic materials) we have

$$|\mathbf{B}| = \mu_0 \mu_r |\mathbf{H}|, \quad \mu_r = 1000$$

- This means for a given field **H**, iron concentrates the flux lines by a factor of 1000 better than air (μ_r = 1).
- Of course, this is only valid if **H** is sufficiently low, **H** < H_{sat}
- For **H** > H_{sat}, the material cannot handle any more flux and no increase in the flux density occurs
  - the material is said to be saturated
  - linear assumption does not hold anymore
  - Constant permeability needs to be replaced by a differential permeability

$$\mu = \frac{\partial \mathbf{B}}{\partial \mathbf{H}}$$

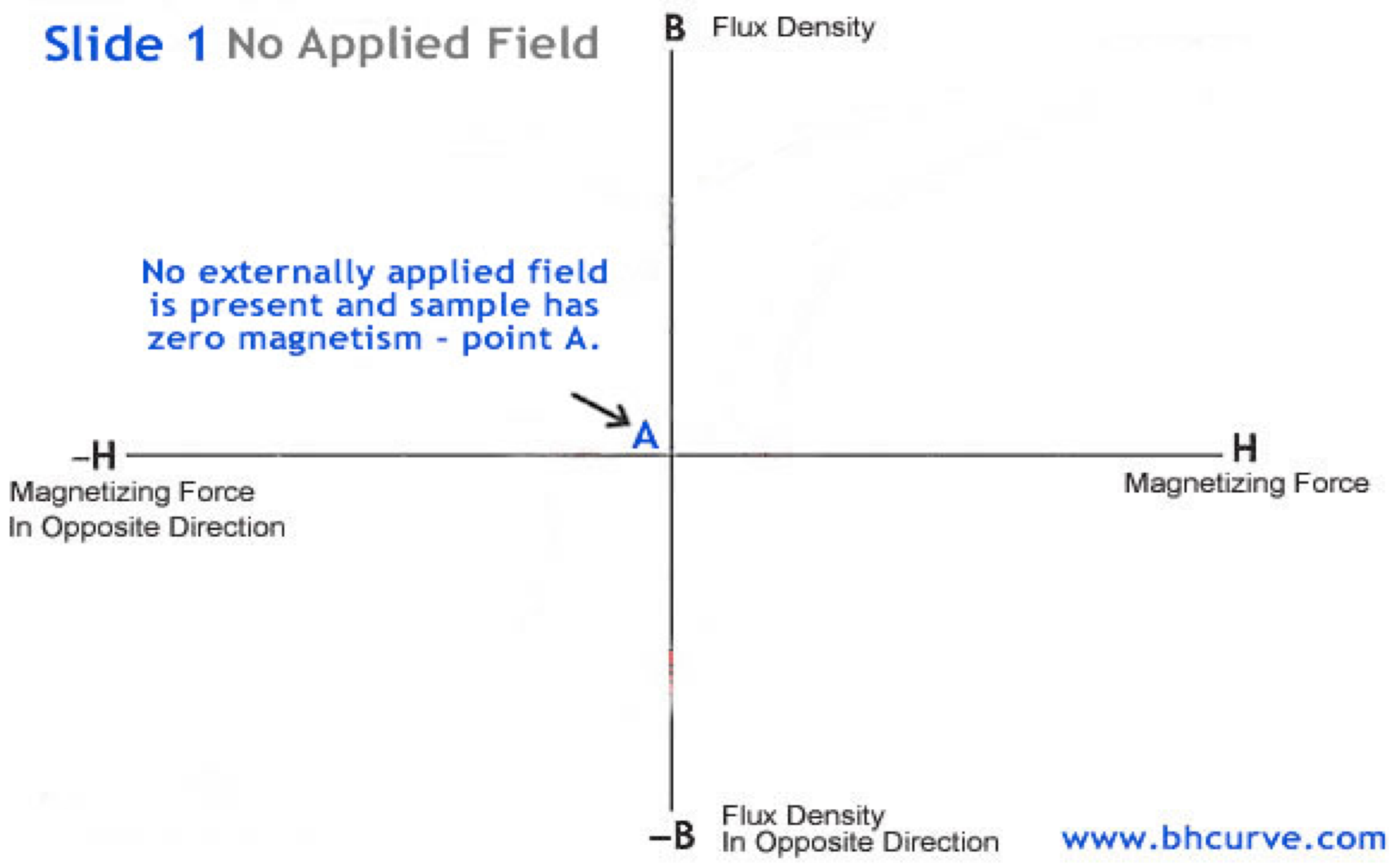
Hysteresis Loop

- The B-H loop defines magnetic quality of material and the best uses
  - **Strong permanent magnet**: high remanence
  - **Memory storage**: high coercivity (stability) and remanence (SNR)
  - **Transformers**: soft with small hysteresis loop (energy loss in AC)
  - **Electromagnets**: soft (linear behavior) and high permeability and saturation

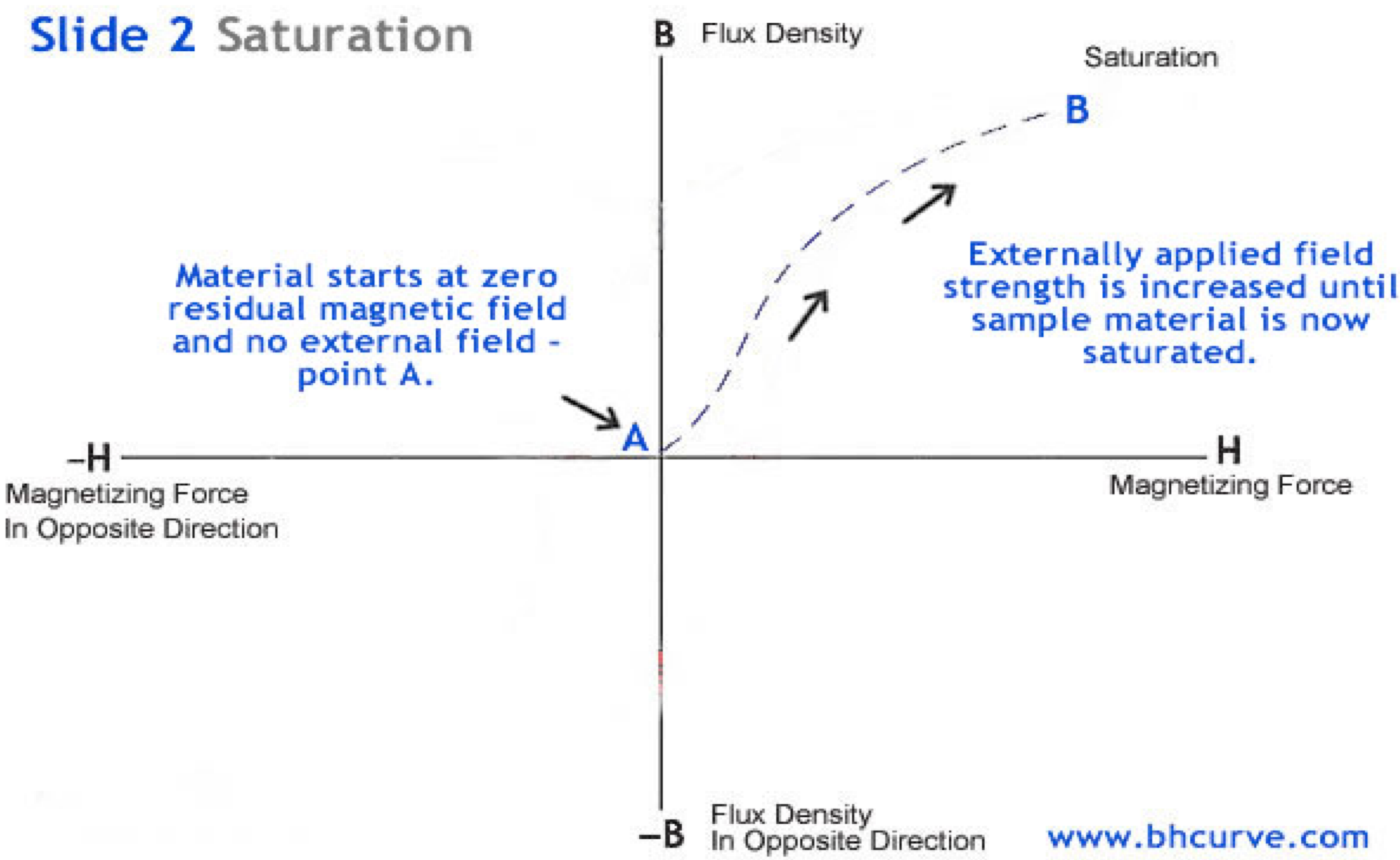
- The permeability μ is not enough to classify magnetic materials
  - Changes depending on the location in the hysteresis loop

- Magnetization **M** is introduced
  - Captures the magnetic state of the material

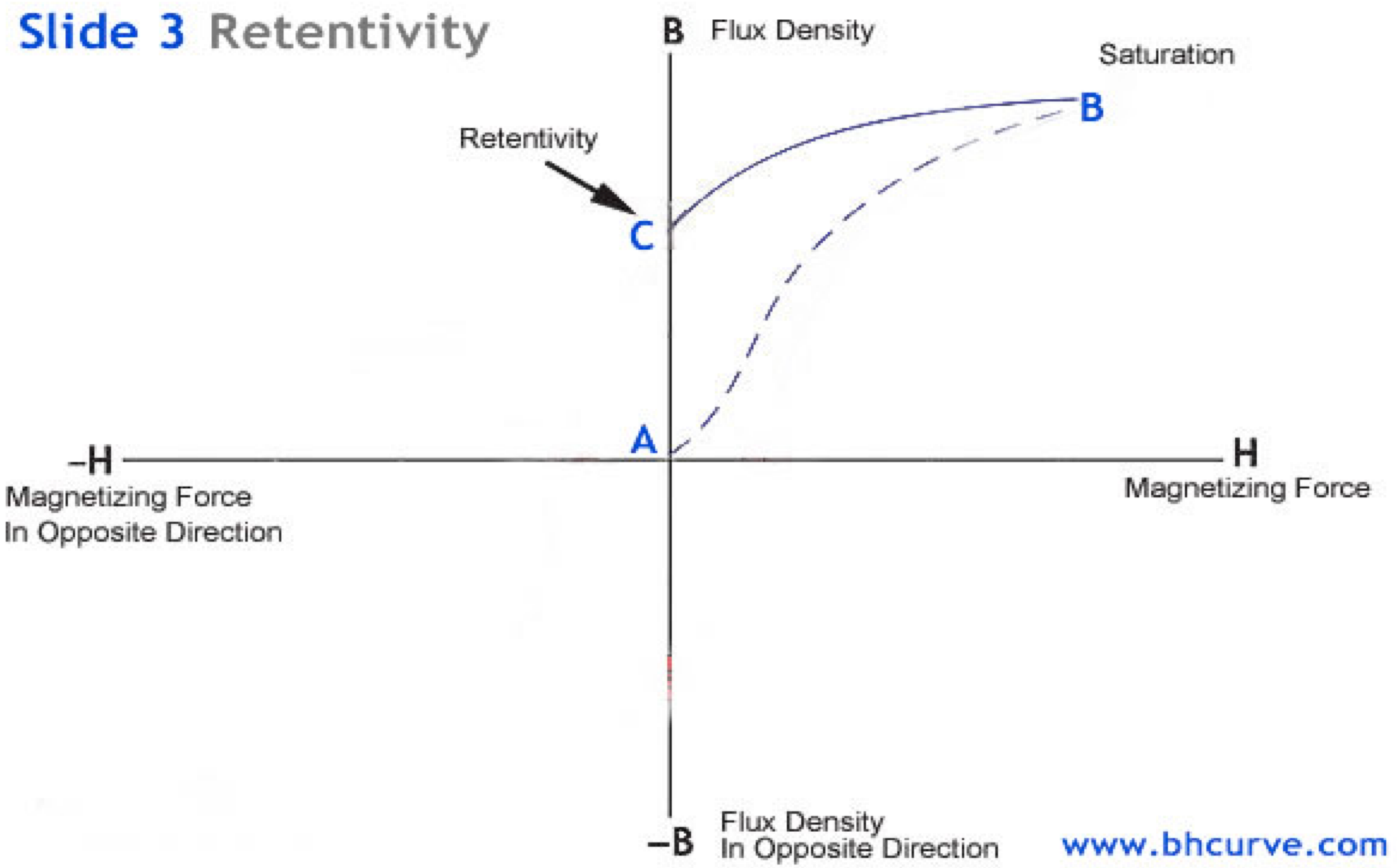
Hysteresis Loop



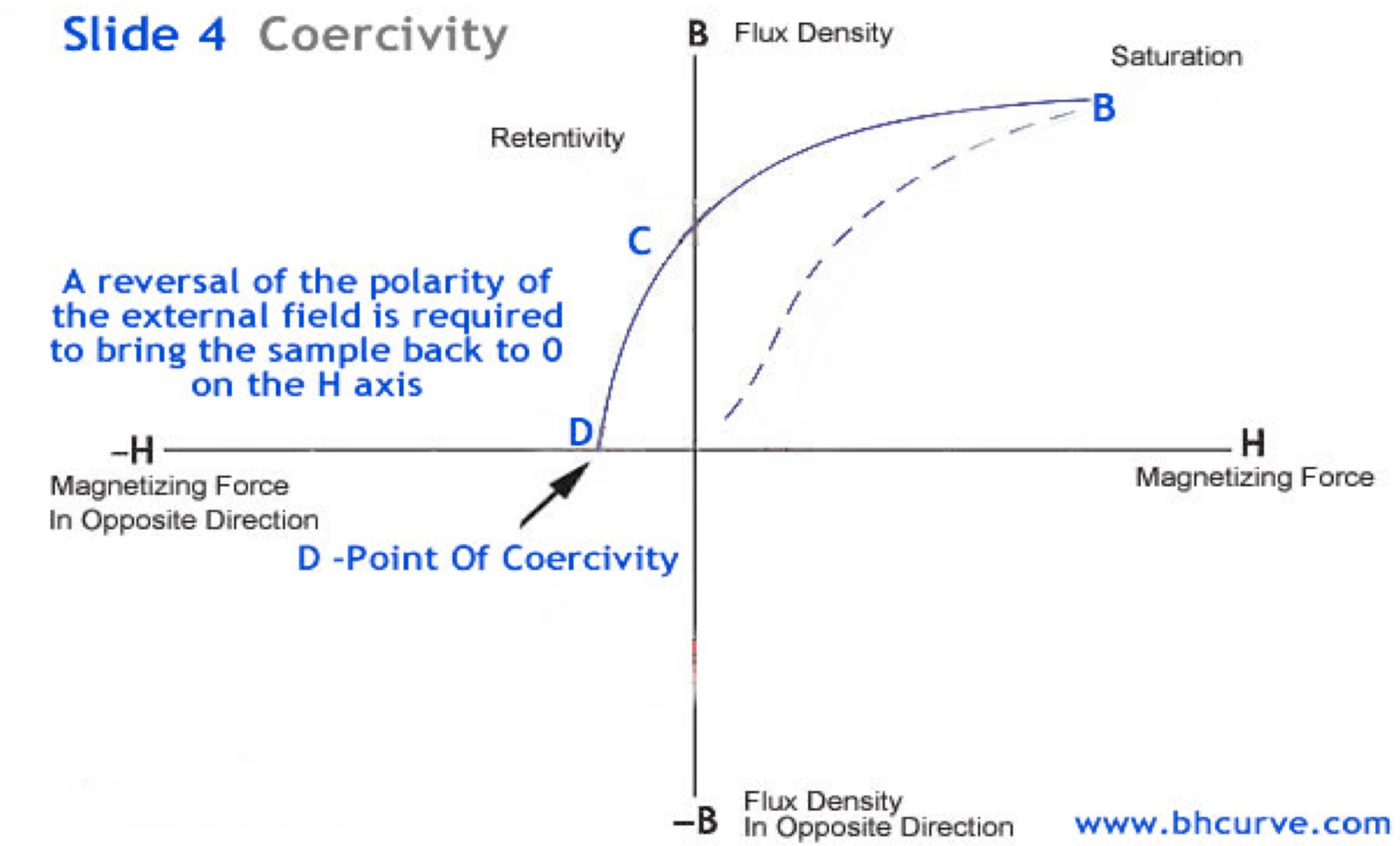
Hysteresis Loop



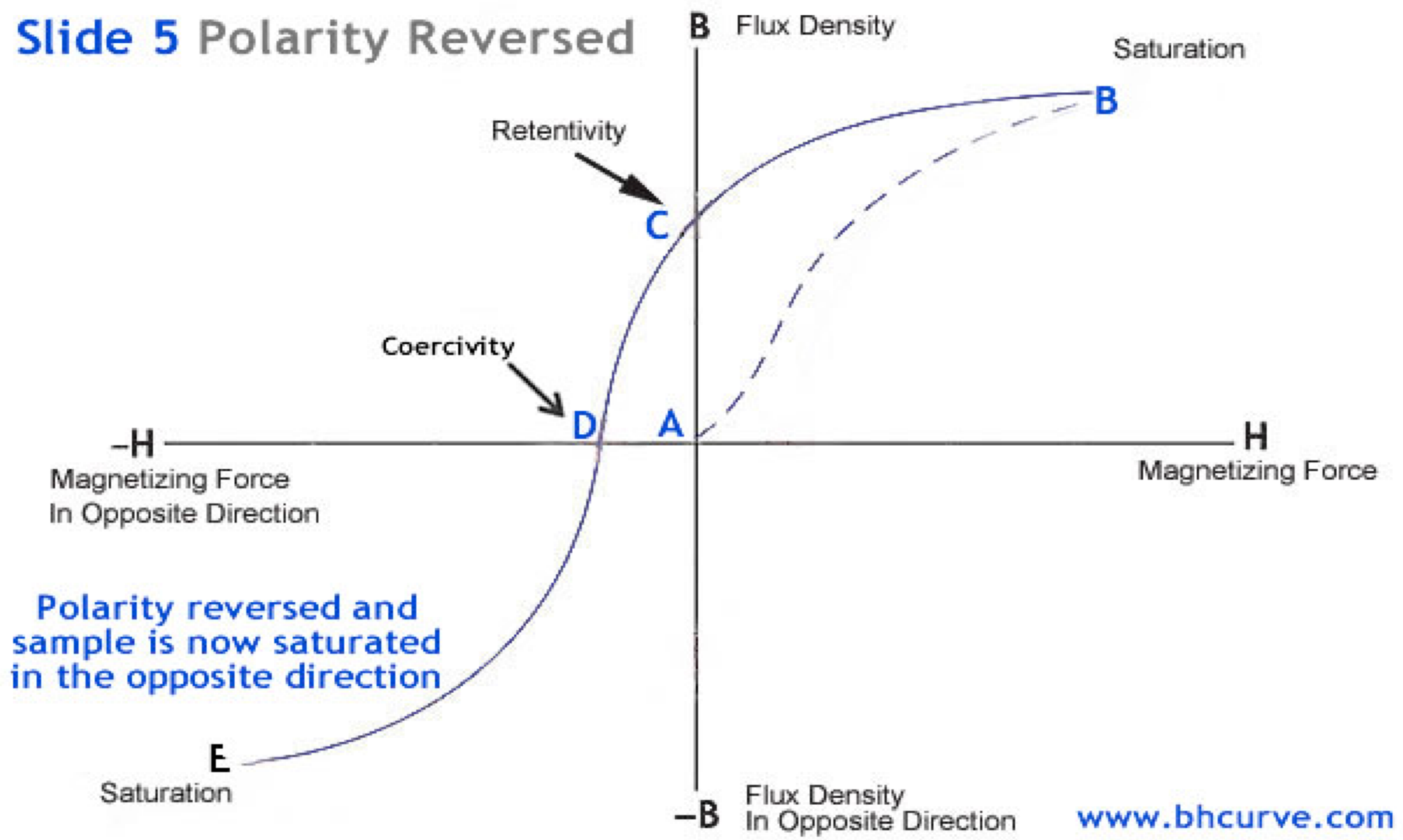
Hysteresis Loop



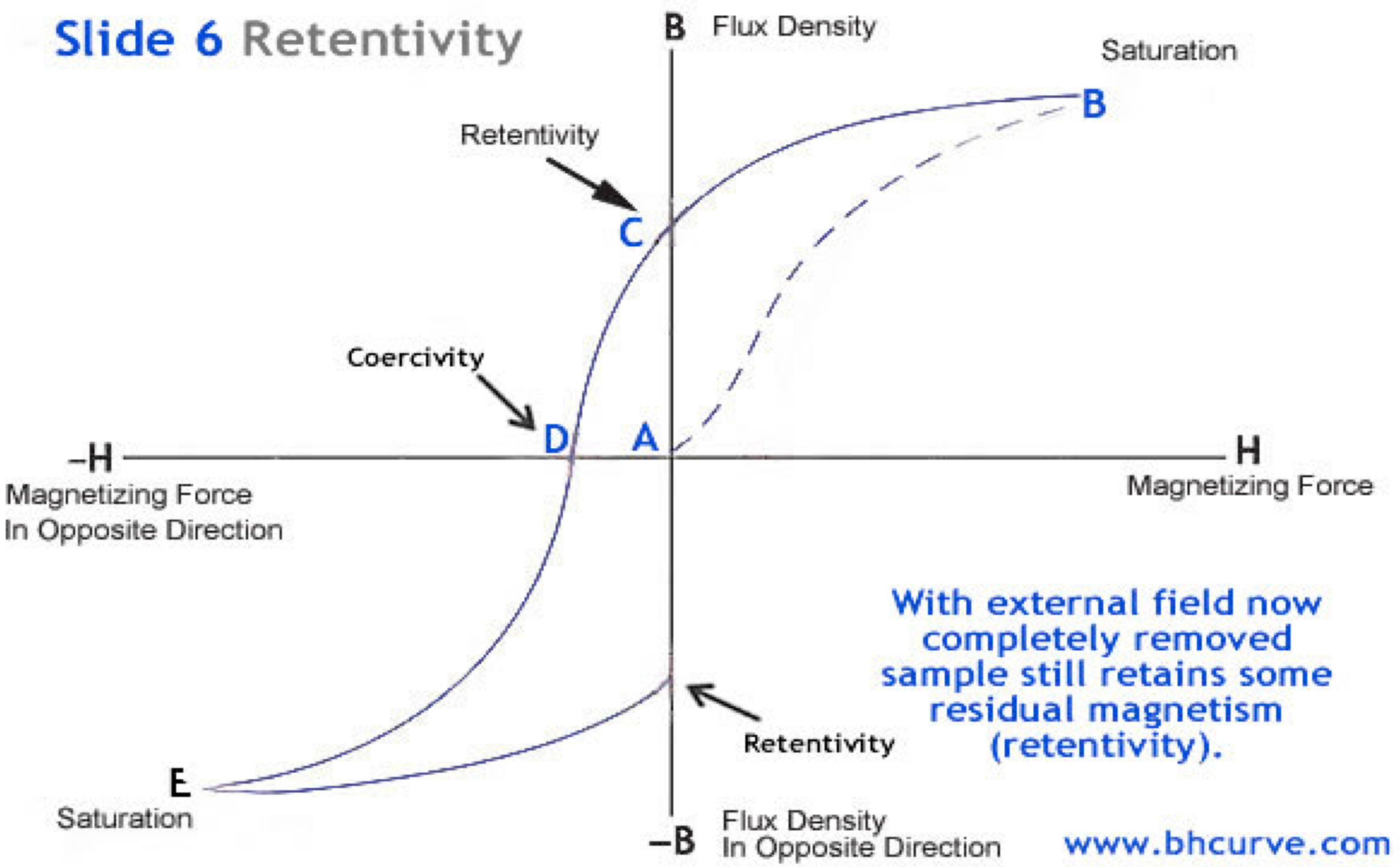
Hysteresis Loop



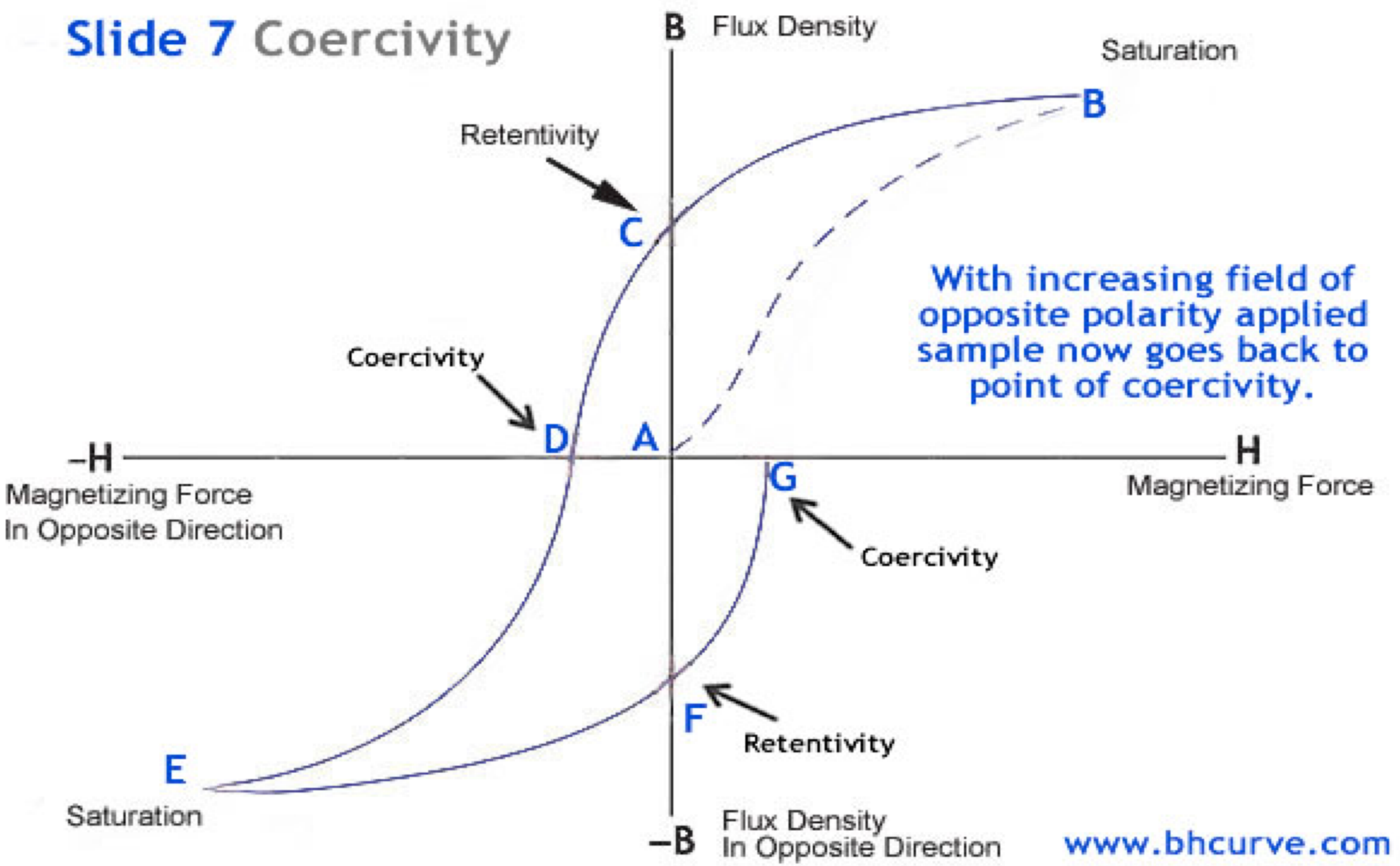
Hysteresis Loop



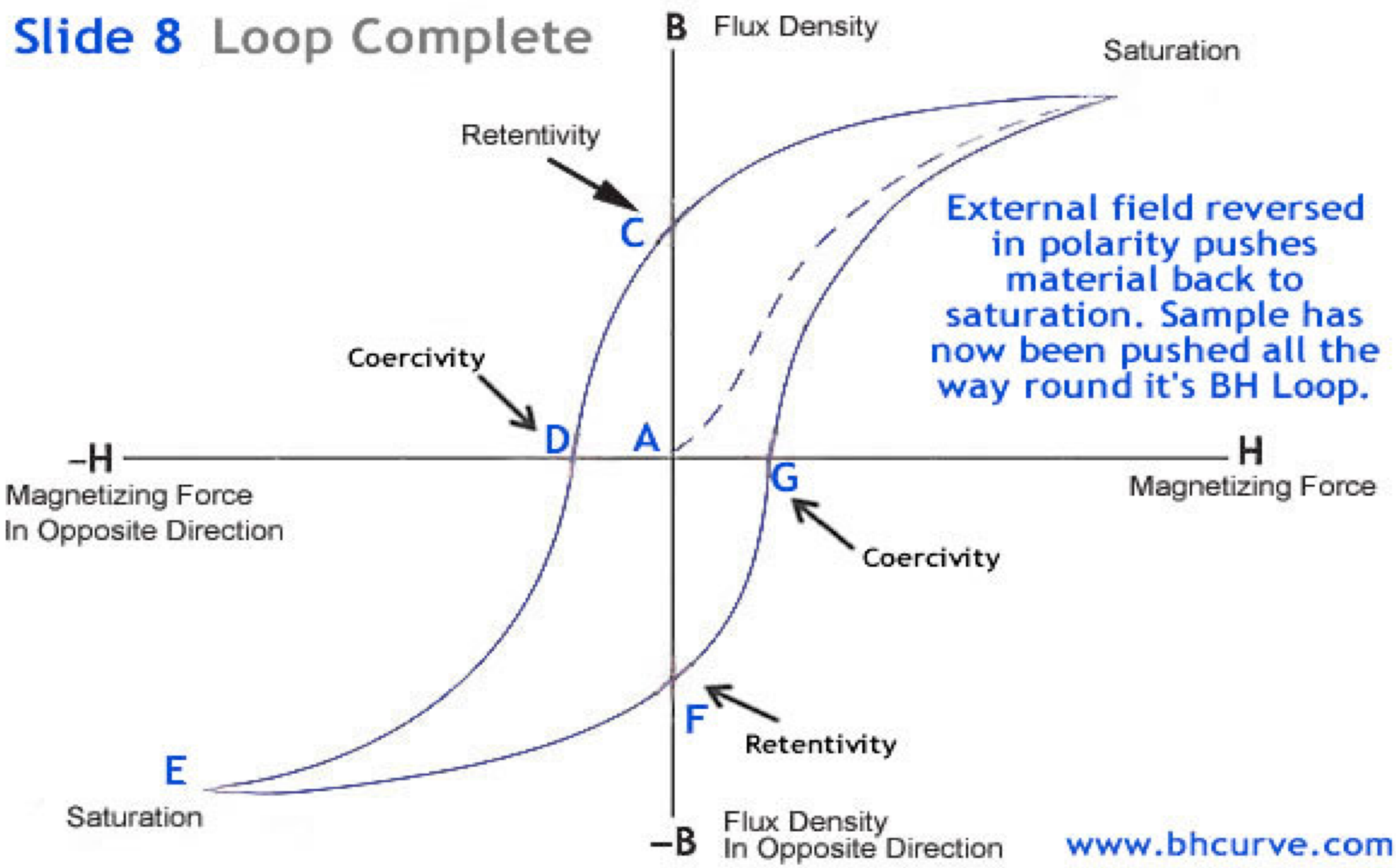
Hysteresis Loop



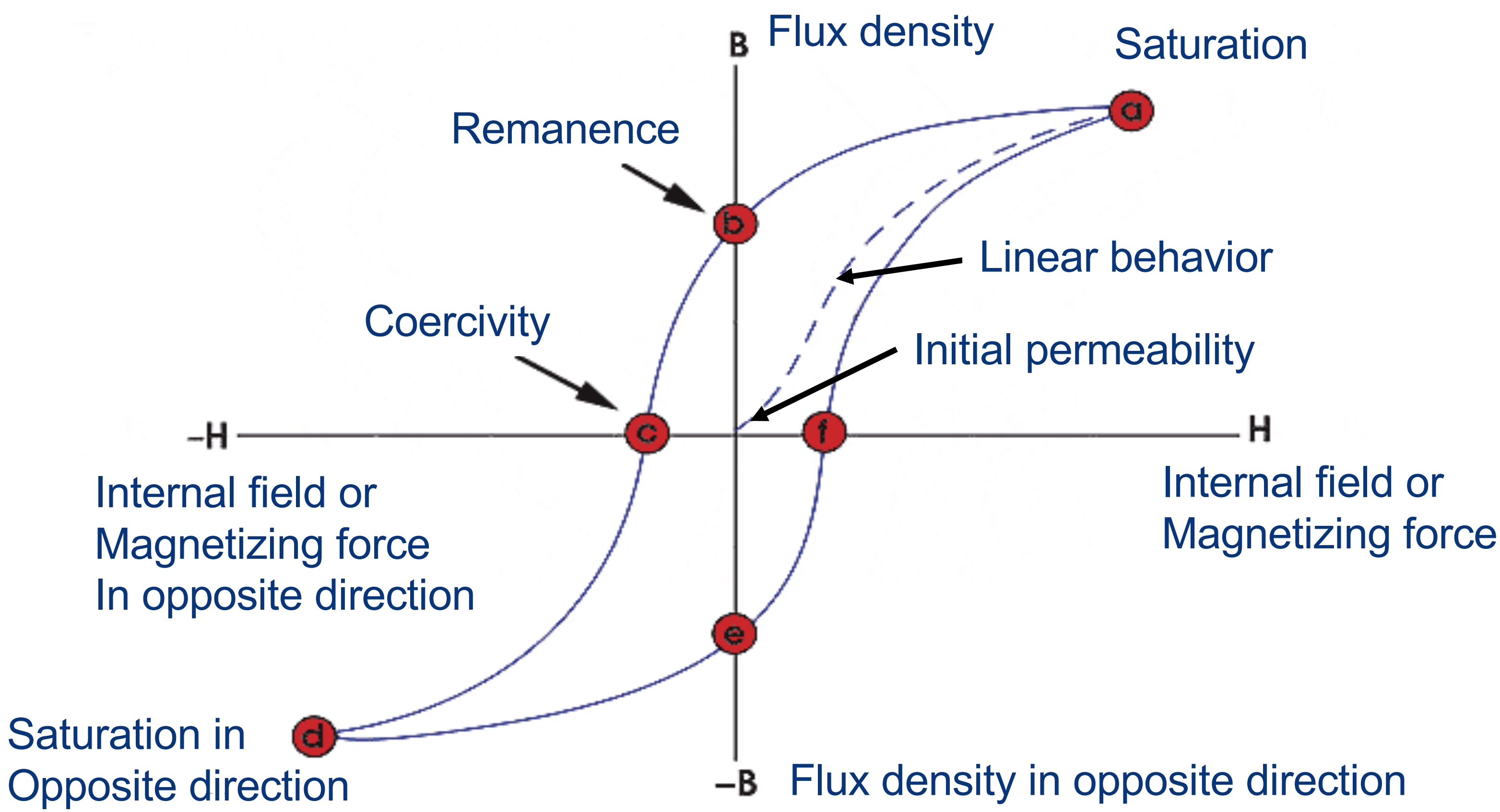
Hysteresis Loop



Hysteresis Loop



Hysteresis Loop

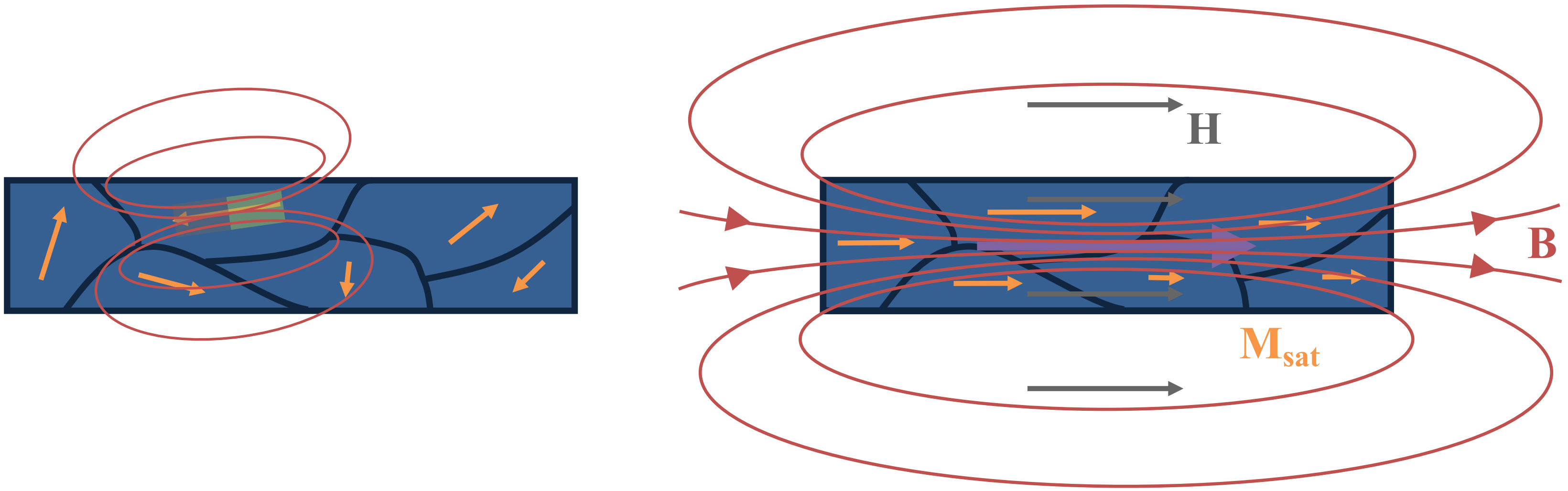


Remanence and Coercivity

- They depend on sample shape, surface roughness, microscopic defects, and thermal history
- Also depends on the rate at which the field is swept in order to trace the hysteresis loop

Magnetization

- What happens when a ferromagnetic material is placed in a sufficiently strong magnetic field?
  - The domain walls move
  - The domains reorient in parallel with the applied field and a net field is generated. If all domains are oriented, the saturation occurs. Increasing the applied field further has no effect anymore.
  - The phenomenon that the material undergoes when it is placed in the magnetic field is called magnetization (and denoted by the vector **M**).



Magnetization

- The behavior when the external field is turned off leads to further classification:
  - In soft magnetic materials, the domain walls will again reorient at random orientations and no or a weak net field is observed
  - In hard magnetic materials, the domain walls will remain reoriented, creating a permanent magnet.

- Magnetization is a vector field **M** = M (x,y,z) (as are **B** and **H**)

$$\begin{aligned}\mathbf{B} &= \mu_0(\mathbf{H}+\mathbf{M}) \\ &= \mu_0\mathbf{H}+\mu_0\mathbf{M}\end{aligned}$$

- In soft magnetic materials, M and H are related by the susceptibility tensor:

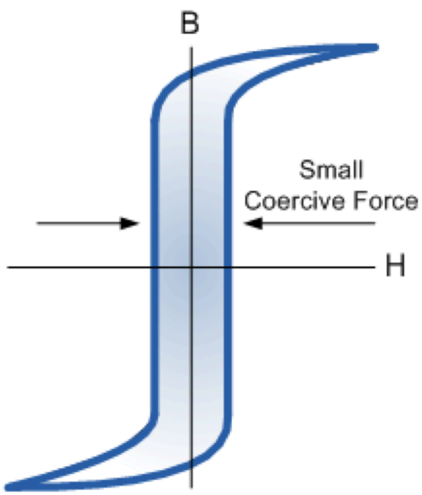
$$\mathbf{M} = \chi\mathbf{H}$$

- After saturation:

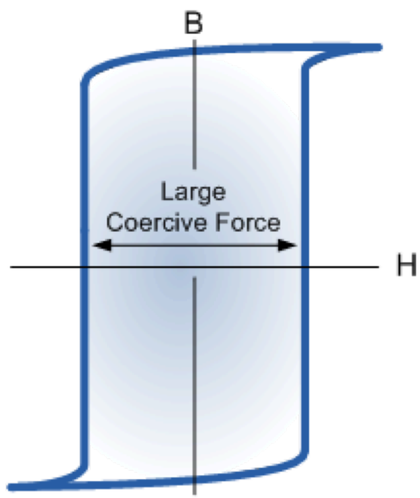
$$|\mathbf{M}| = m_s$$

Magnetization

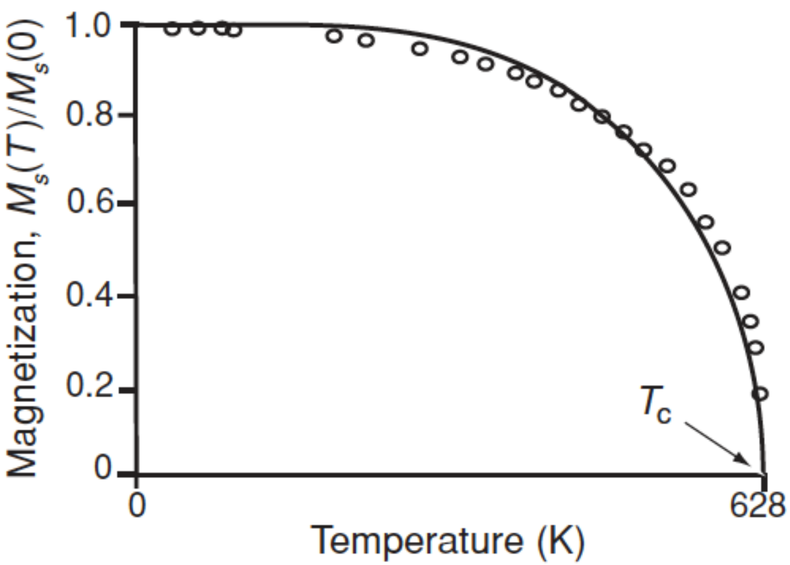
- Spontaneous magnetization due to alignment of the atomic magnetic moments depends on temperature
- Random alignment at the Curie temperature
- Curie temperature of iron is 1044K, cobalt 1388K, nickel 628K



"Soft" Ferromagnetic Material



"Hard" Ferromagnetic Material

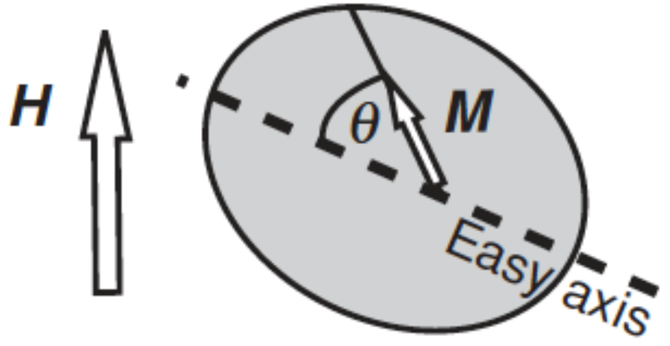


Easy Axis

- The natural direction of magnetization is usually constrained to lie along one or more easy axes
- This tendency is represented by the anisotropy energy

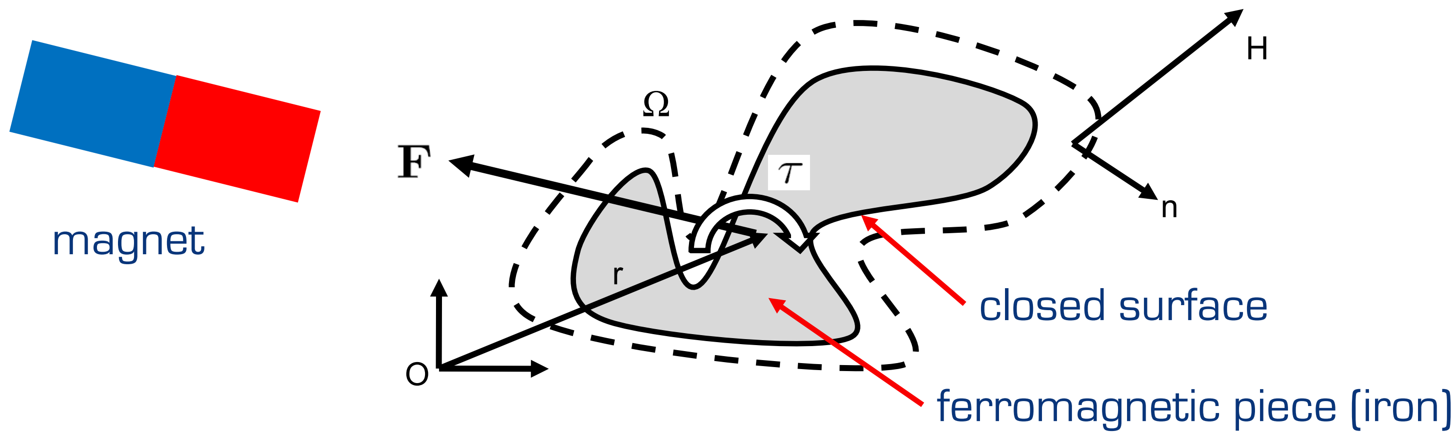
$$E_a = K sin^2\theta$$

- Shape matters: Magnetization of the object is not necessarily parallel to the applied field



Maxwell Stress Tensor

- The forces and torques around the body



The force and torque can be computed as

$$\mathbf{F} = \oint_{\Omega} \mathbf{T} \cdot \mathbf{n} d\Omega$$

$$\tau = \oint_{\Omega} [\mathbf{r} \times (\mathbf{T} \cdot \mathbf{n})] d\Omega$$

Where T is the Maxwell stress tensor:

$$\mathbf{T} = \mathbf{H} \otimes \mathbf{B} - \frac{1}{2}(\mathbf{H} \cdot \mathbf{B})\mathbb{1}$$

- This method requires knowledge of the magnetic field close to the surface of the body
- Widely used in finite element software

Approximation

- The forces and torques around the body

$$\mathbf{F} = \mu_0 \int_v (\mathbf{M}\cdot\nabla)\mathbf{H}\,\mathrm{d}\nu \approx \mu_0 v(\mathbf{M}\cdot\nabla)\mathbf{H}$$

$$\tau = \mu_0 \int_v (\mathbf{M}\times\mathbf{H})\,\mathrm{d}\nu \approx \mu_0 v\mathbf{M}\times\mathbf{H}$$

Applied (external) field

- In the integral expressions, **M** and **H** generally vary over the body.
- **Simpifications**:
- 1. Consider **H** only at the center of mass of the body
  - Valid only when the body is small compared to spatial changes in the applied field
- 2. Treat **M** as a single vector.
  - Only truly valid for ellipsoids
  - Body is now treated as a magnetic dipole

Magnetization

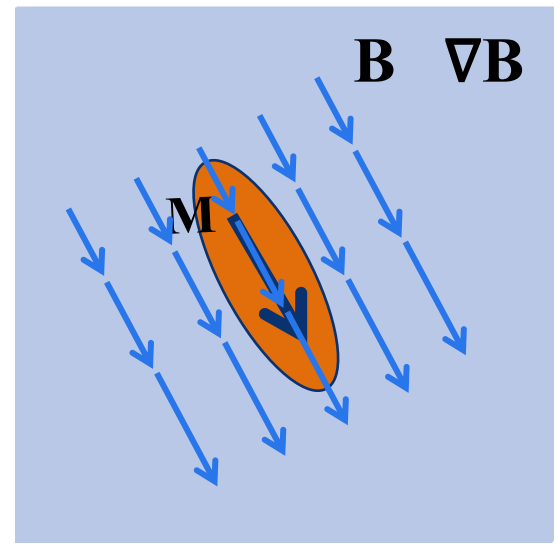
- For small bodies, e.g. microrobots, we assume:
  - Uniform distribution of the applied field **B** throughout the body
  - **M** is a single vector (body is viewed as a dipole)

- Magnetic Torque: $\mathbf{T} = \upsilon\, \mathbf{M} \times \mathbf{B}$

- Magnetic Force: $\mathbf{F} = \upsilon\, (\mathbf{M} \cdot \nabla)\, \mathbf{B}$

$$\nabla \times \mathbf{B} = 0$$

$$\mathbf{F} = v \begin{bmatrix} \frac{\partial \mathbf{B}}{\partial x} & \frac{\partial \mathbf{B}}{\partial y} & \frac{\partial \mathbf{B}}{\partial z} \end{bmatrix}^T \mathbf{M}$$



Force and Torque on Hard Magnets

- Consider the force between two identical magnets with magnetization **M** and volume **V**, aligned along their dipole axes, and separated by distance **x**.

- The field created by one magnet along its axis

$$H(x) = \frac{MV}{2\pi x^3}$$

- The magnitude of the attractive/repulsive force on the other magnet

$$F = \mu_0 MV \left|\frac{\partial H}{\partial x}\right| = \frac{3\mu_0 M^2 V^2}{2\pi x^4}$$

- If we scale the magnets down as $\sim L$ but hold the distance constant, the force will be scaled as $\sim L^6$ (poor scaling). If we also scale the distance, the force is scaled as $\sim L^2$.

Scaling Analysis (electrostatics)

- Assuming that all dimensions scale linearly proportional to L
- Volume scales with L^3
  - Inertia, weight, heat capacity, and body forces
- Surface area scales with L^2
  - Friction, heat transfer, surface forces

- Assuming that the voltage and d is constant, F scales as $\sim L^2$
- If we scale the gap as well then F remains the same

$$F = \frac{\partial W}{\partial d} = \frac{\varepsilon A V^2}{2d^2}$$

Force and Torque on Soft Magnets

- The magnetization vector is **dependent** on the applied field and the equations are nonlinear and iterative

- The magnetization will change to minimize the magnetic energy of the system. This is not a practical approach for analytical results. Therefore, we decouple the problem
  1. The applied field induces magnetization in the material
  2. The induced magnetization interacts with the applied field

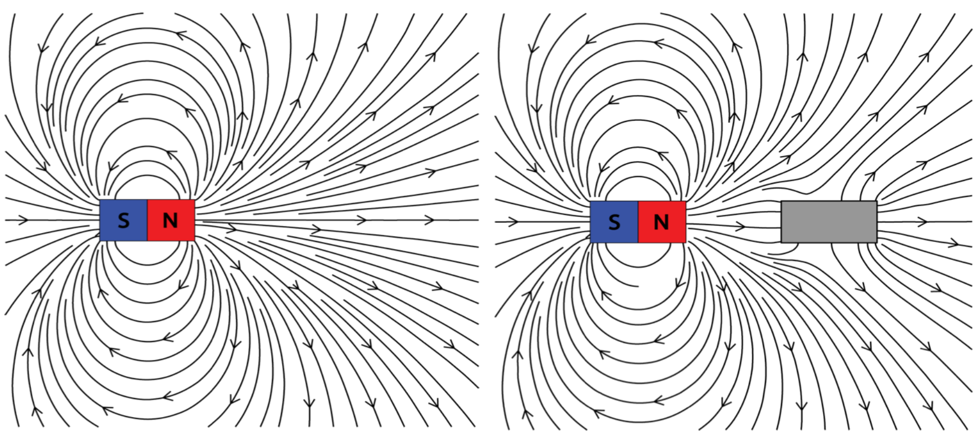
Magnetic Field Generation

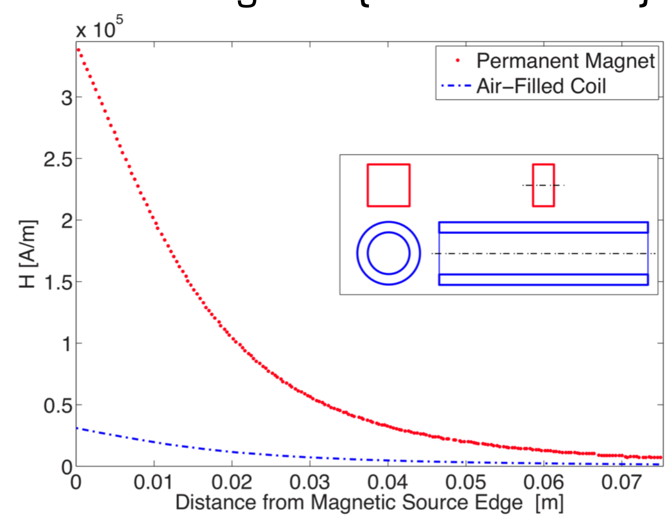
Permanent Magnets

+ Strong field on a per volume basis
+ Miniaturization
– Interaction between adjacent magnets
– Actuation required to control the field strength
– Magnets always on, special shielding mechanism needed to turn field off

Electromagnets

+ Linear superposition of individual field contributions with air core
+ Field strength can be adjusted through current
+ Field can "immediately" be switched on and off
– Week field strength compared to permanent magnets (for same size)

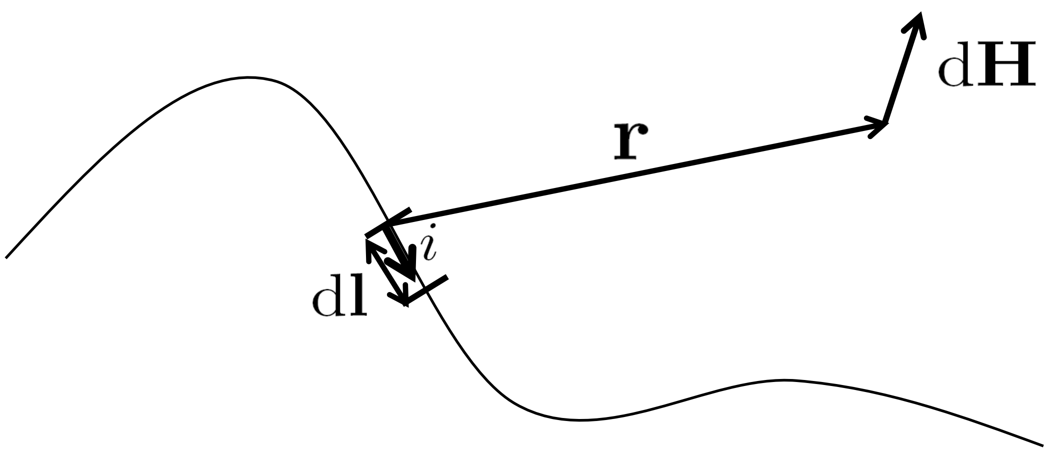




Biot-Savart Law

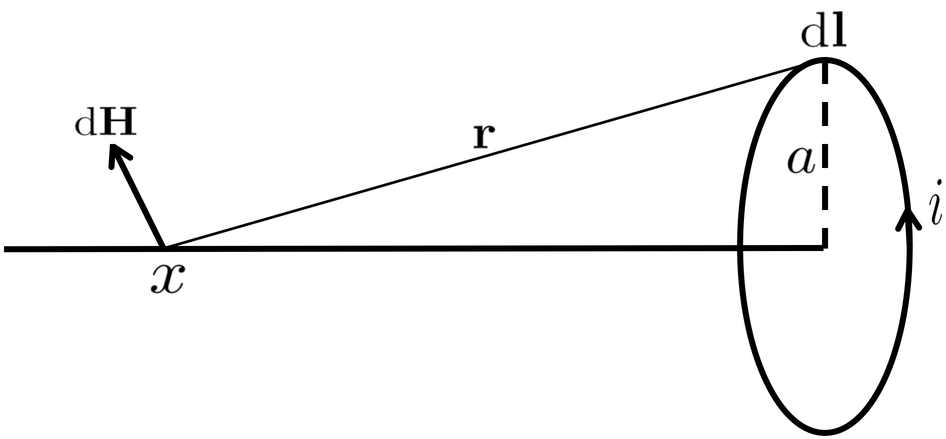
- Calculates the magnetic field **H** generated by an electrical current i:

$$\mathrm{d}\mathbf{H} = \frac{1}{4\pi|\mathbf{r}|^2} i \mathrm{d}\mathbf{l} \times \hat{\mathbf{r}}$$



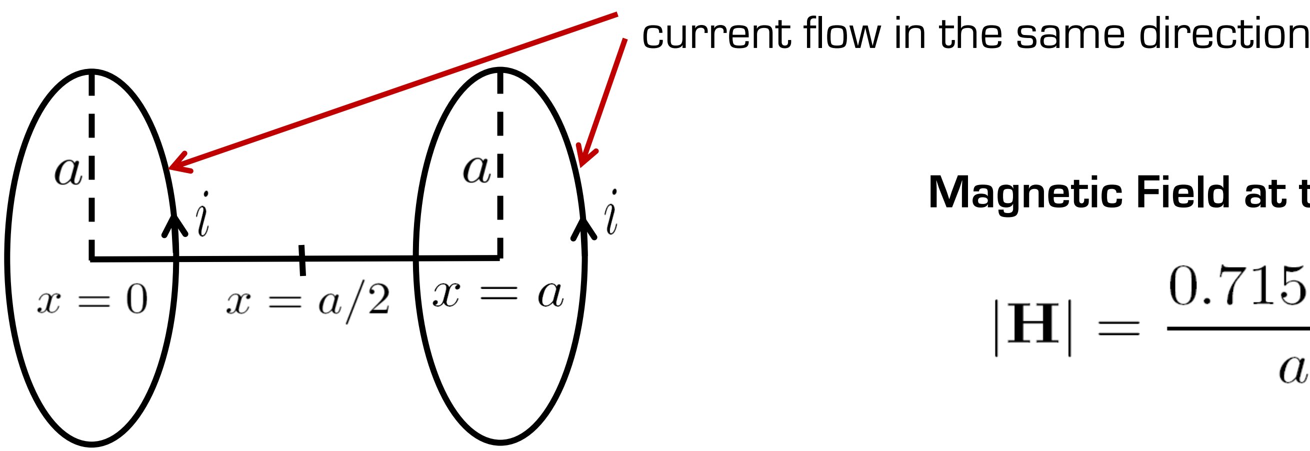
Helmholtz Coil

- Magnetic field **H** at a distance x along the centerline:



$$|\mathbf{H}| = \frac{ia^2}{2(a^2+x^2)^{3/2}}$$

$$= \left(\frac{i}{2a}\right)\left(1+\frac{x^2}{a^2}\right)^{-1.5}$$

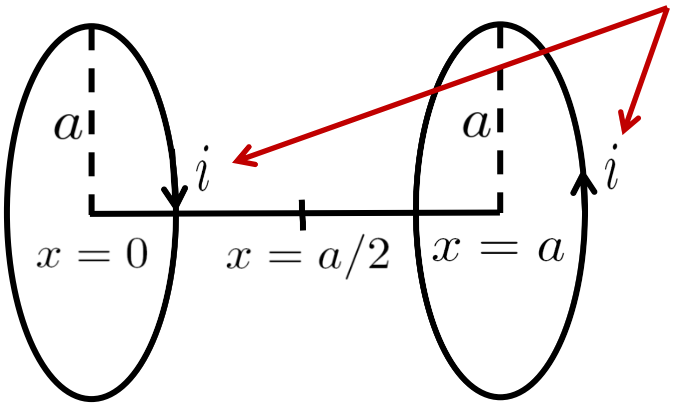


Magnetic Field at the center

$$|\mathbf{H}| = \frac{0.7155Ni}{a}$$

Maxwell Coil

- Uniform magnetic field gradient in the center

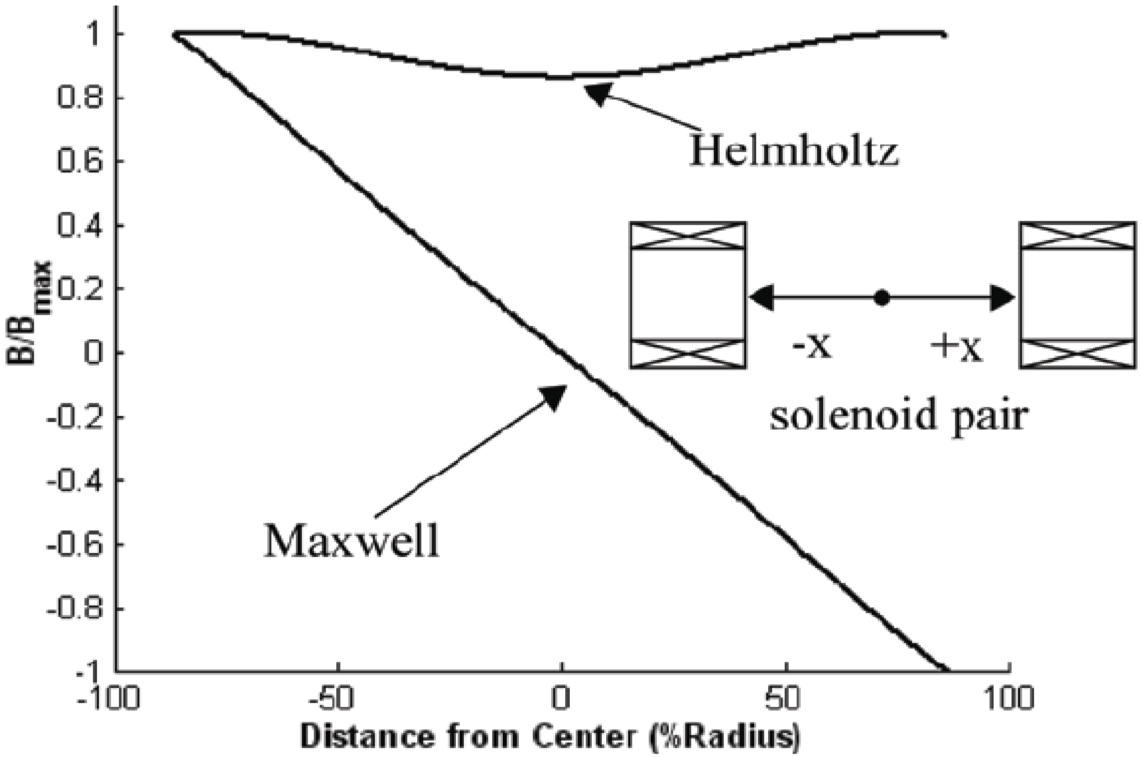


Magnetic Field at the center

$$|\mathbf{H}| = \left(\frac{Ni}{2a}\right)\left[\left(1+\frac{x^2}{a^2}\right)^{-1.5} - \left(1+\frac{(a-x)^2}{a^2}\right)^{-1.5}\right] = 0$$

Helmholtz vs Maxwell Configuration

- Uniform magnetic field vs magnetic field gradient in the center



Solenoid

- Field along axis of a solenoid
  - Length L, Diameter D, N turns, centered around x = 0

$$|\mathbf{H}| = \left(\frac{Ni}{L}\right)\left[\frac{L+2x}{2\left[D^2+(L+2x)^2\right]^{1/2}} + \frac{L-2x}{2\left[D^2+(L-2x)^2\right]^{1/2}}\right]$$

- At the center (x = 0), this simplifies to

$$|\mathbf{H}| = \left(\frac{Ni}{L}\right)\left[\frac{L}{(L^2+D^2)^{1/2}}\right]$$

- For long solenoids (L>>D), this simplifies further to

$$|\mathbf{H}| = \frac{Ni}{L}$$

- The field at the end of the solenoid (x = L/2) is half the value of the field at the center
- The field in the middle of the solenoid is uniform

Control with Stationary Electromagnets

- Within a given static arrangement of electromagnets, every electromagnet creates a field throughout the workspace that can be pre-computed

- Field magnitude at a given point P can be expressed as a unit-current vector [T/A] multiplied by a scalar current value [A]:

$$\mathbf{B}_e(\mathbf{P}) = \tilde{\mathbf{B}}_e(\mathbf{P}) i_e$$

e : denotes e[th] electromagnet

- $\mathbf{B}_e(\mathbf{P})$: Field due to current flowing through electromagnet e and due to soft-magnetic cores in all other electromagnets

Control with Stationary Electromagnets

- Assumptions:
  - Field contribution of a given electromagnet was precomputed
  - Use of soft-magnetic core material with negligible hysteresis
  - Operation of the system with the cores in their linear magnetization region
- Field contributions of the individual currents (each of which affect the magnetization of every core) superimpose linearly

$$\mathbf{B}(\mathbf{P}) = \sum_{e=1}^{n} \mathbf{B}_e(\mathbf{P}) = \sum_{e=1}^{n} \tilde{\mathbf{B}}_e(\mathbf{P}) i_e$$

$$\mathbf{B}(\mathbf{P}) = \begin{bmatrix} \tilde{\mathbf{B}}_1(\mathbf{P}) & \cdots & \tilde{\mathbf{B}}_n(\mathbf{P}) \end{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} = \mathcal{B}(\mathbf{P}) I$$

Control with Stationary Electromagnets

- The derivative of the field in a given direction in a specific frame can also be expressed as the contributions from each of the currents:

$$\frac{\partial \mathbf{B}(\mathbf{P})}{\partial x} = \begin{bmatrix} \frac{\partial \tilde{\mathbf{B}}_1(\mathbf{P})}{\partial x} & \cdots & \frac{\partial \tilde{\mathbf{B}}_n(\mathbf{P})}{\partial x} \end{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} = \mathcal{B}_x(\mathbf{P}) I$$

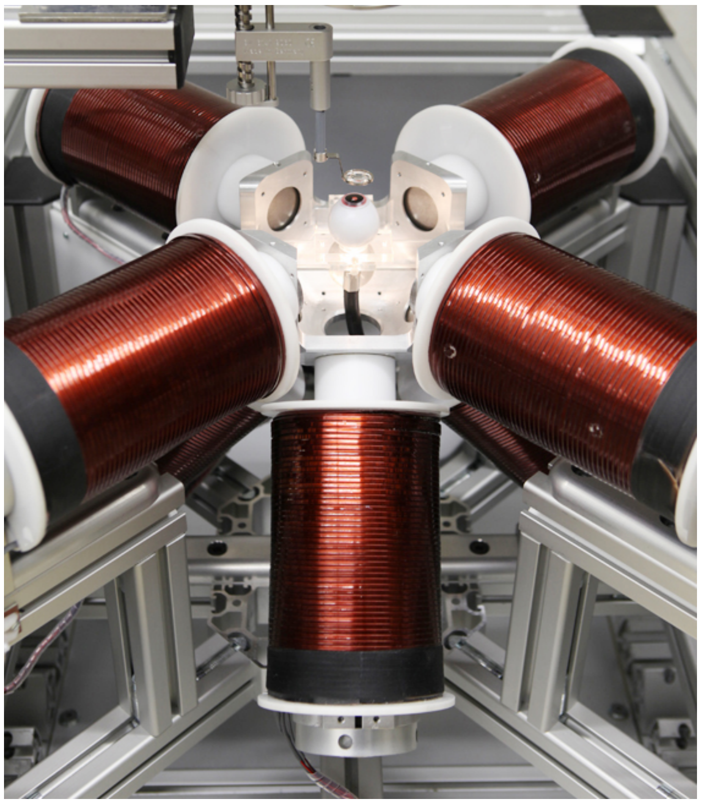
$$\frac{\partial \mathbf{B}(\mathbf{P})}{\partial z} = \begin{bmatrix} \frac{\partial \tilde{\mathbf{B}}_1(\mathbf{P})}{\partial z} & \cdots & \frac{\partial \tilde{\mathbf{B}}_n(\mathbf{P})}{\partial z} \end{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} = \mathcal{B}_z(\mathbf{P}) I$$

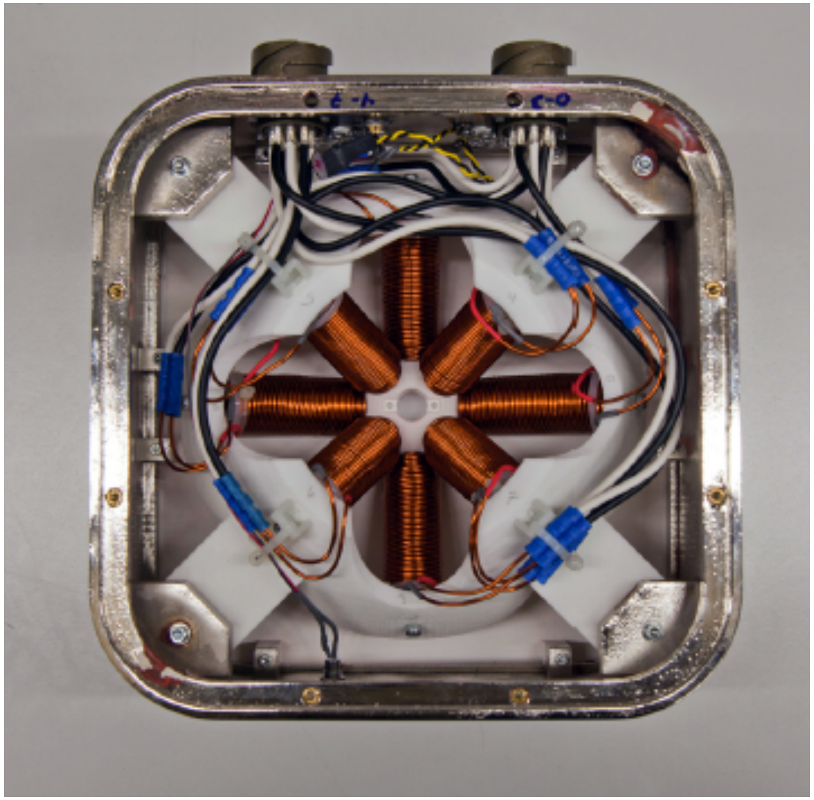
$$\frac{\partial \mathbf{B}(\mathbf{P})}{\partial y} = \begin{bmatrix} \frac{\partial \tilde{\mathbf{B}}_1(\mathbf{P})}{\partial y} & \cdots & \frac{\partial \tilde{\mathbf{B}}_n(\mathbf{P})}{\partial y} \end{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} = \mathcal{B}_y(\mathbf{P}) I$$

$\mathcal{B}(\mathbf{P}), \mathcal{B}_x(\mathbf{P}), \mathcal{B}_y(\mathbf{P}), \mathcal{B}_z(\mathbf{P})$: 3-by-n matrices that are known at each point in the workspace and can be precomputed or calculated online

Electromagnetic Control Systems

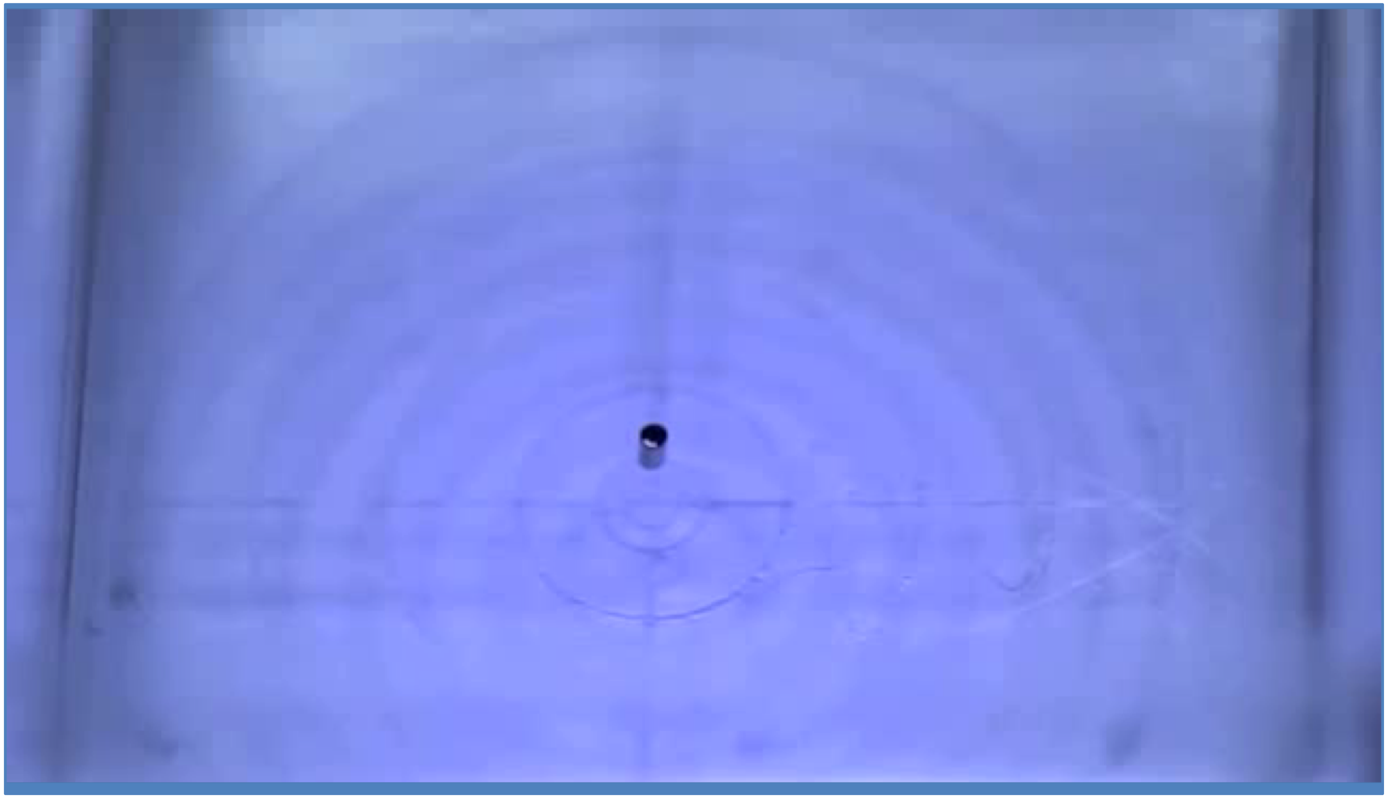
- 8 electromagnetic coils (minimum # required)
- Independent control over orientation and position
- Coil size (and power) vs workspace



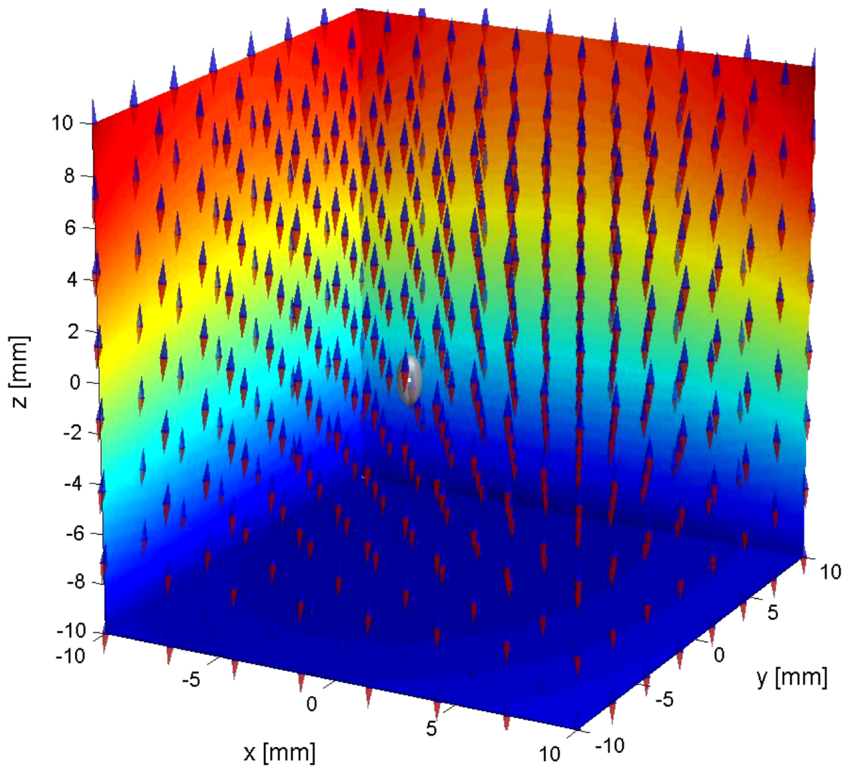


Demonstration of 5-DOF control

- Controlling 6th DOF (roll) is not possible for an object with single magnetization axis



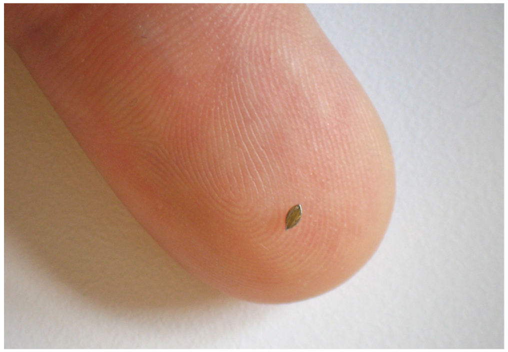
Demonstration of 5-DOF control



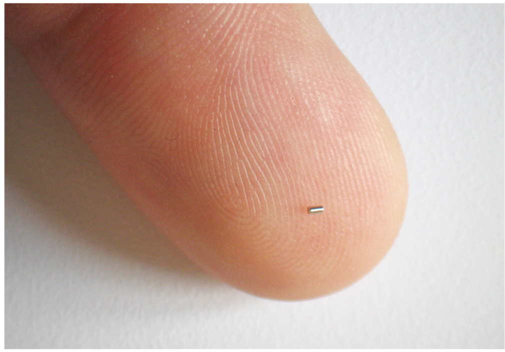
Forces

Device Type	F_{up} (μN)	F_{down} (μN)	$F_{lat,x}$ (μN)	$F_{lat,y}$ (μN)	$F_{lat,xy}$ (μN)
Ni microassembled 500× 250 μm	3.8	3.8	2.1	2.1	3.0
CoNi microassembled 2000× 1000 μm	77.5	77.5	43.5	43.5	61.1
NdFeB cylinder 1000 × 500 μm	287.0	287.0	233.9	233.9	287.0
SmCo tube 2400× 380 μm	365.0	365.0	209.1	209.1	296.3

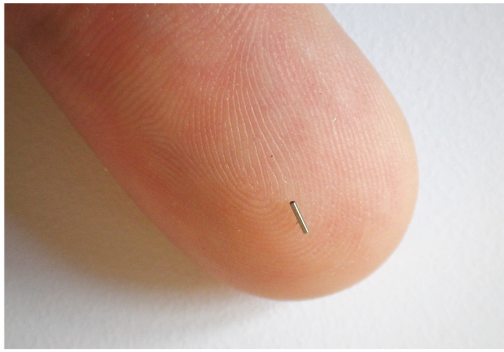
with field aligned along *z* and |B| = 15 mT



2mm CoNi



1mm NdFeB

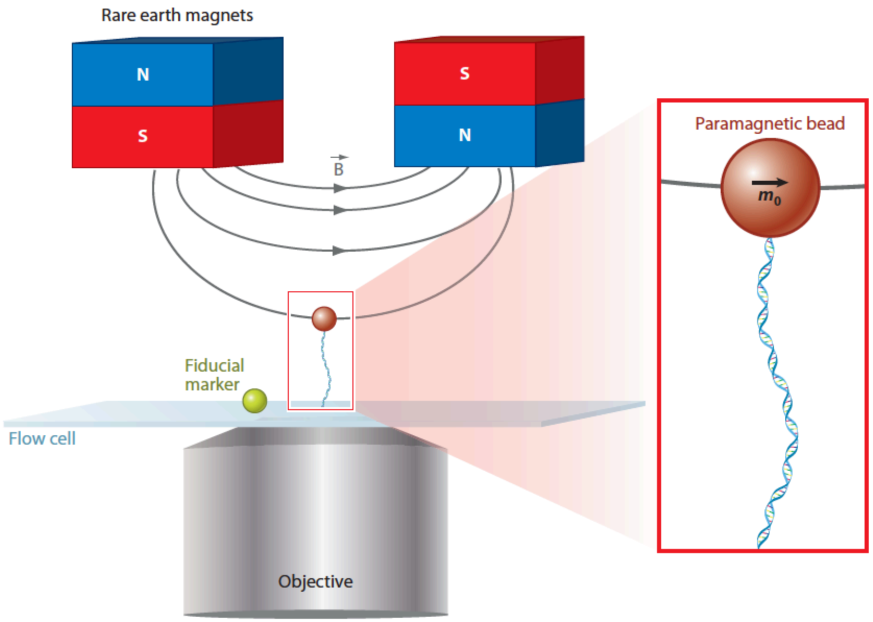


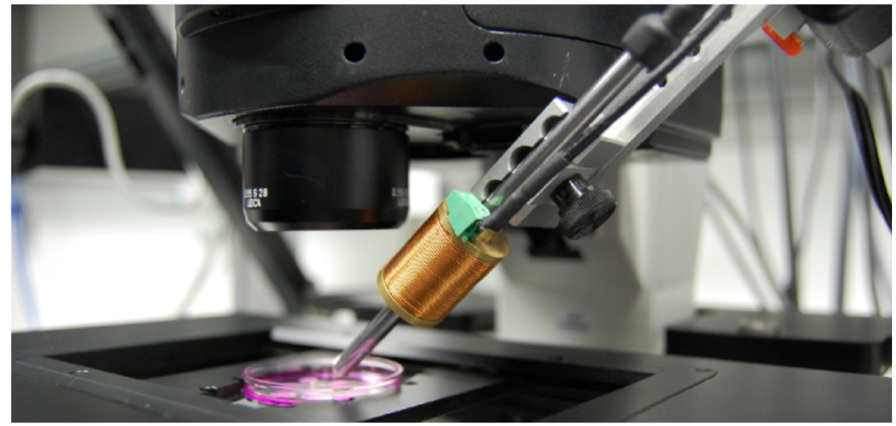
2.4mm SmCo

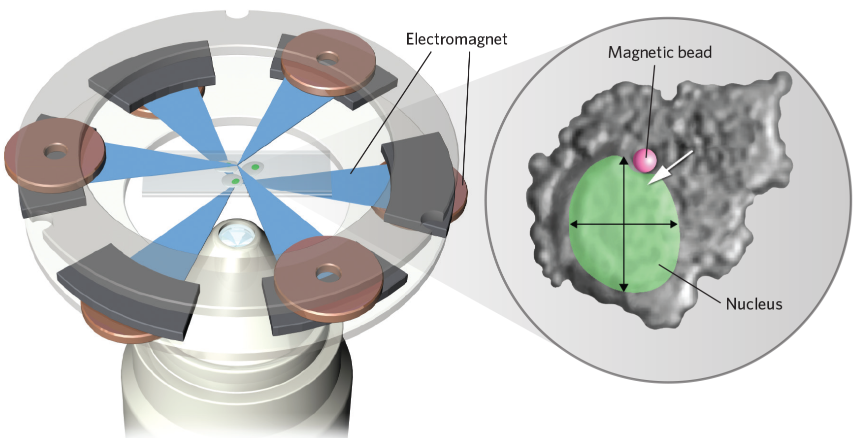
Gradient Pulling

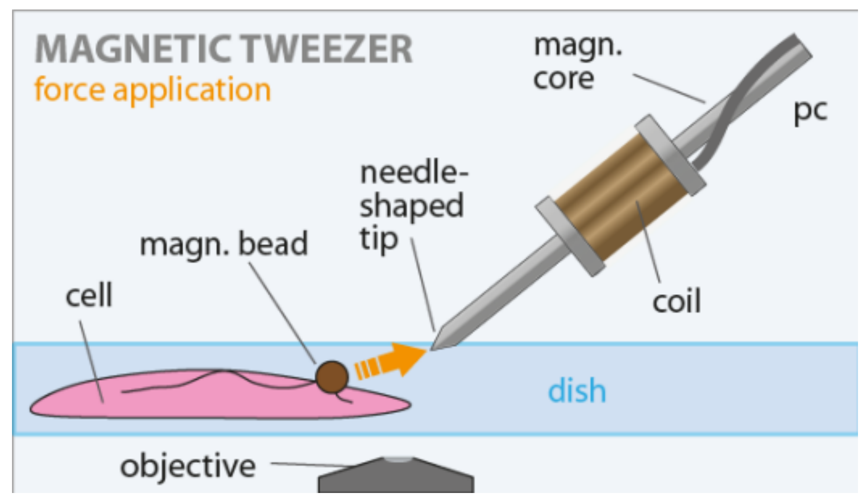
- High Force (1-100 nN)
- Stability
  - Limited workspace
  - Active gradient control
  - Damping: viscous solutions
- Scaling of magnetic forces
- Gravity compensation
  - Increasing buoyancy with air compartments: loss of magnetic volume
  - Lighter materials (i.e. elastomer, hydrogels): magnetic composites
  - Feedback control

Magnetic Tweezers



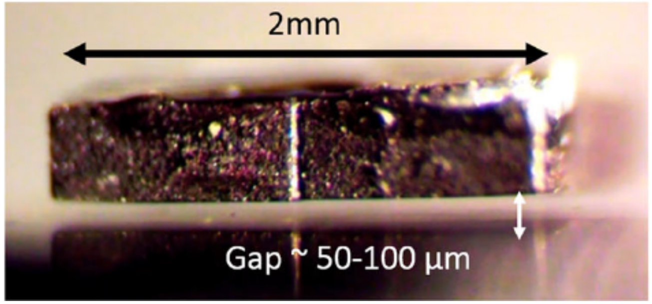


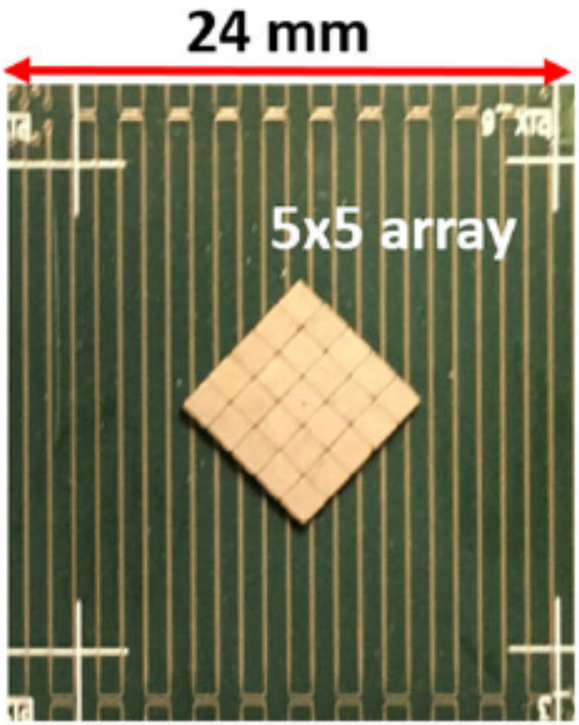


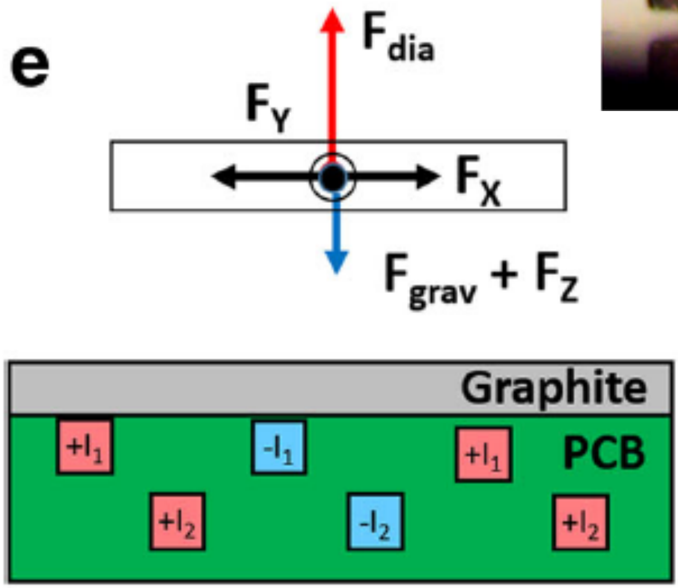


Magnetic Levitation

- Assembled arrays of NdFeB magnets (1.4mm) arranged in an alternating north-south configuration
- Serpentine electrical traces pattern within PCB board
- Two sets of traces for controlling x and y position



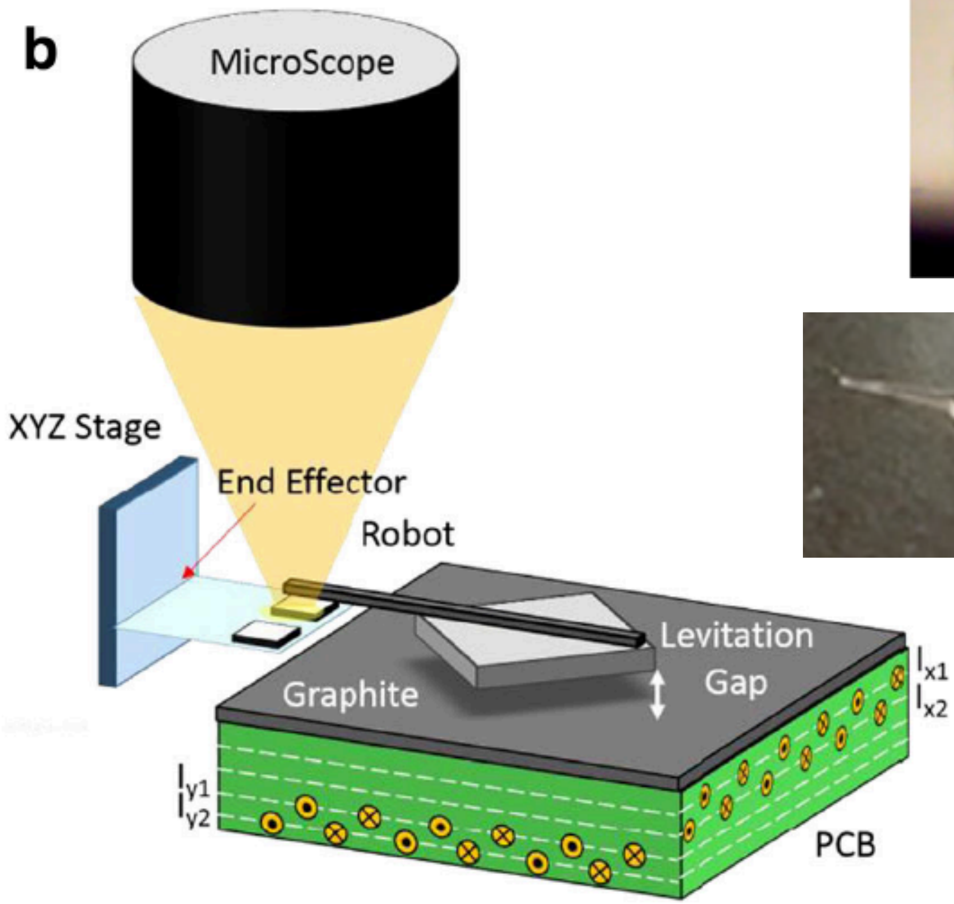


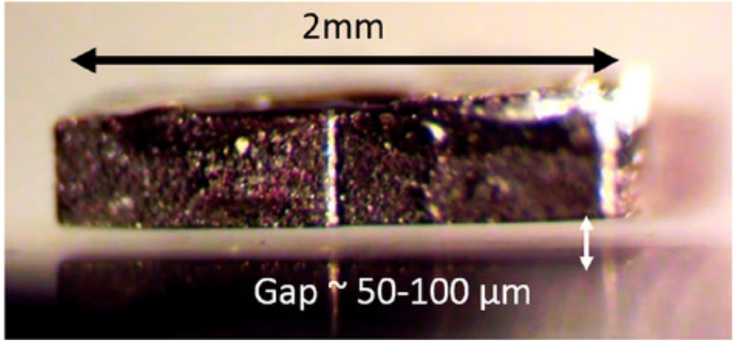


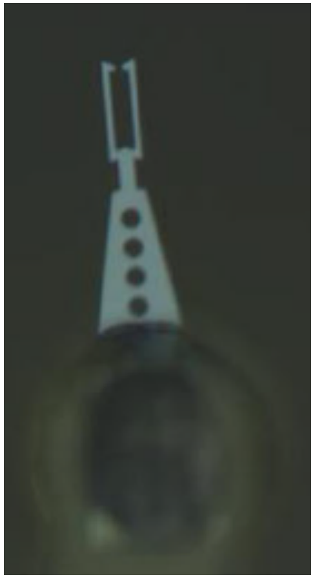
Magnetic Levitation

- SRI Video

- Silicon microtools as end-effectors

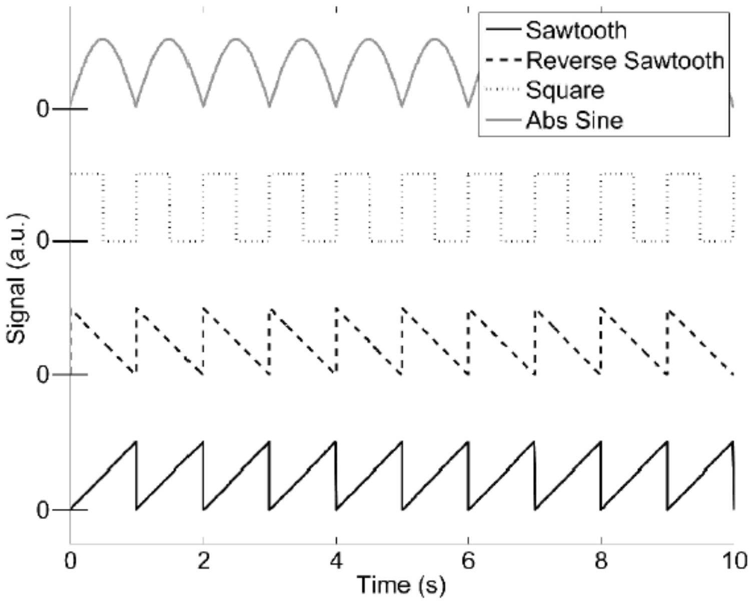


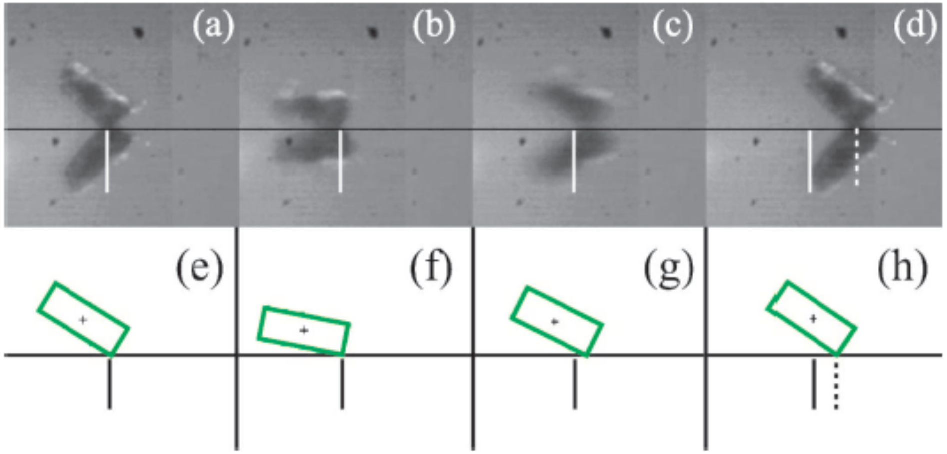


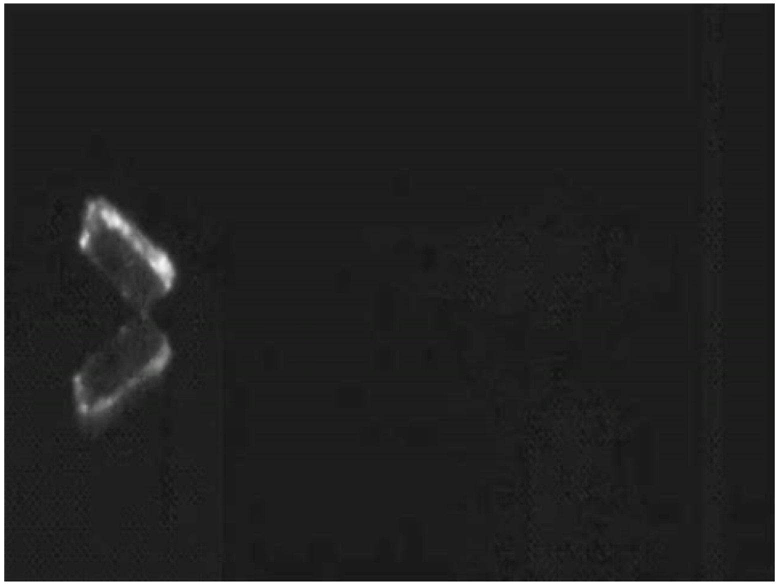


Rocking Motion

In-plane uniform magnetic field
Out-of-plane magnetic torque (pulse)
Asymmetric pulsing waveform
    Stick-slip motion

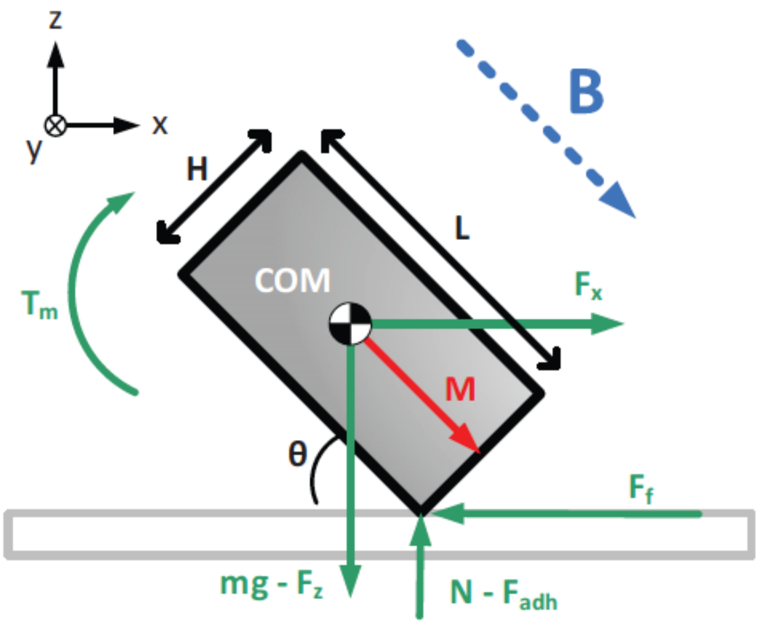






Free Body Diagram

Floyd etal, *IJRR*, 2013



Magnetic Forces (F_X and F_Z)
Magnetic Torque (T_Y)
Damping Force (L_X and L_Y)
Rotational Damping Force (D_Y)
Coulomb Friction Force (F_f)
Adhesion (F_{adh})

Magnetic Forces are negligible (nN)
Magnetic Torque dominates motion (μN)

$$m\ddot{x} = F_x - F_f - L_x,$$

$$m\ddot{z} = F_z - mg + N - F_{\text{adh}} - L_z,$$

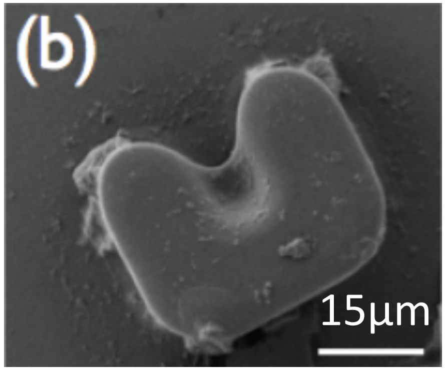
$$J\ddot{\theta} = T_y + F_f \cdot r \cdot \sin(\theta + \phi) - (N - F_{\text{adh}})r \cdot \cos(\theta + \phi) - D_y$$

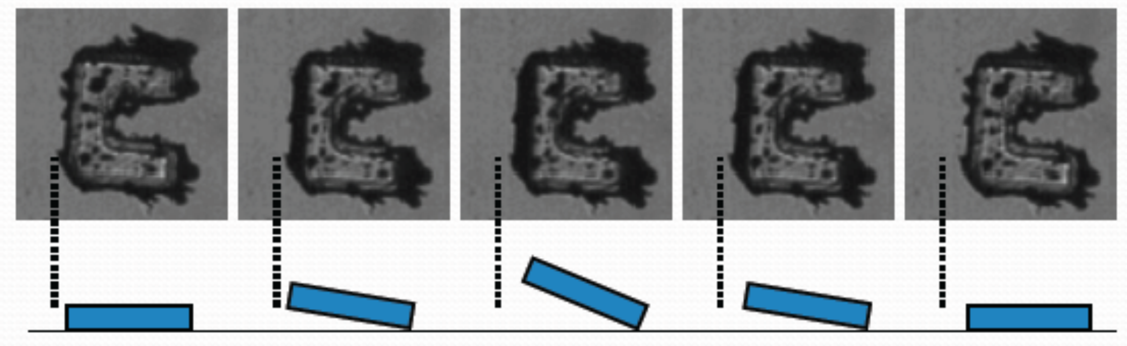
$$J = m(H^2 + L^2)/12.$$

Gravitational Rest Torque (ρ: density)

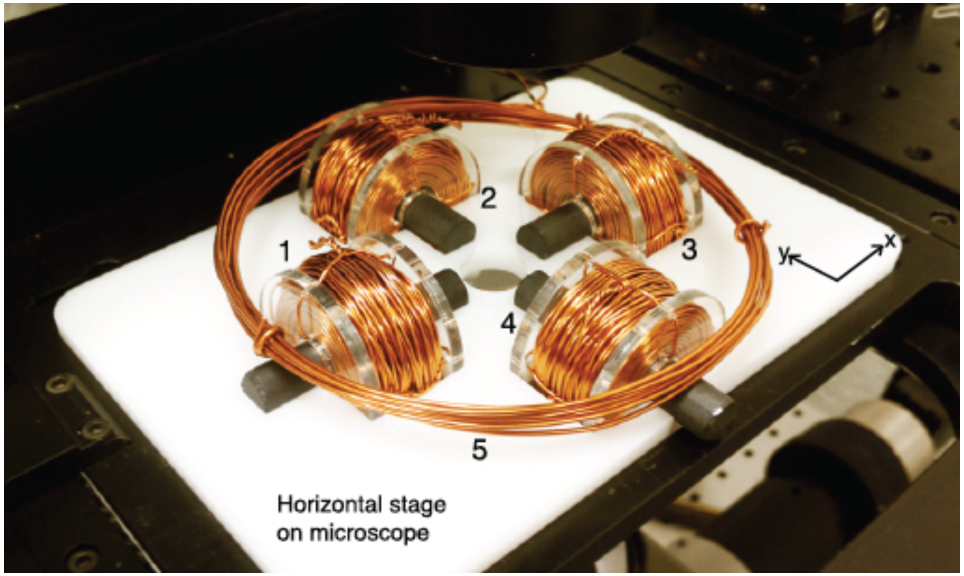
$$T_g = \rho_{\text{eff}} V_m g \frac{L}{2} \qquad \rho_{\text{eff}} = \rho - \rho_{\text{fluid}}$$

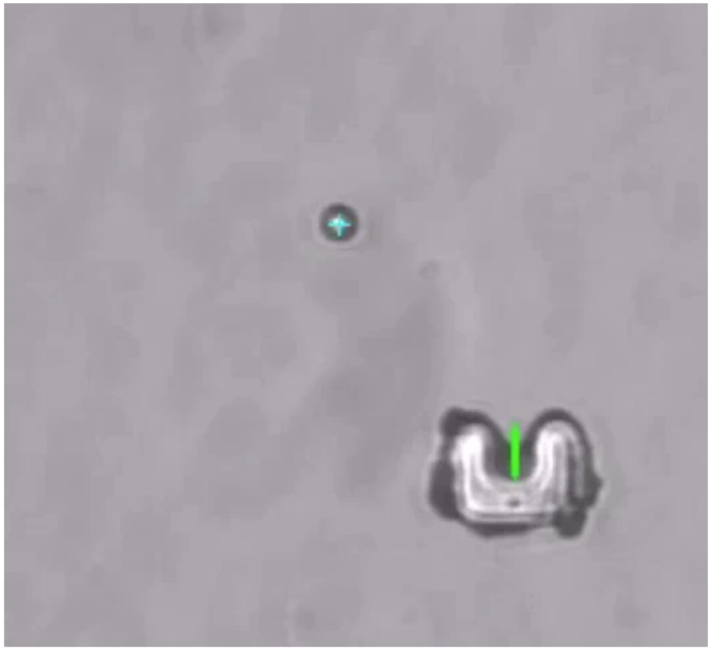
Microtransporters



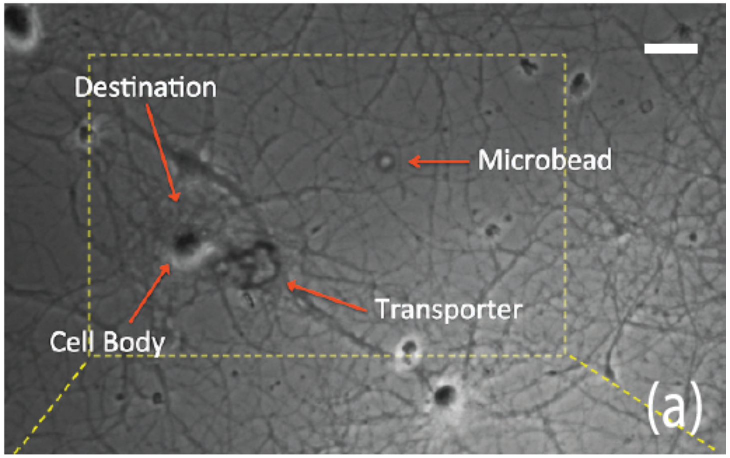


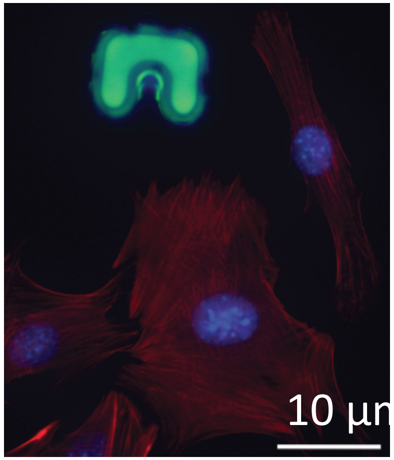
In-plane magnetic force and out-of-plane magnetic torque (clamping coil)



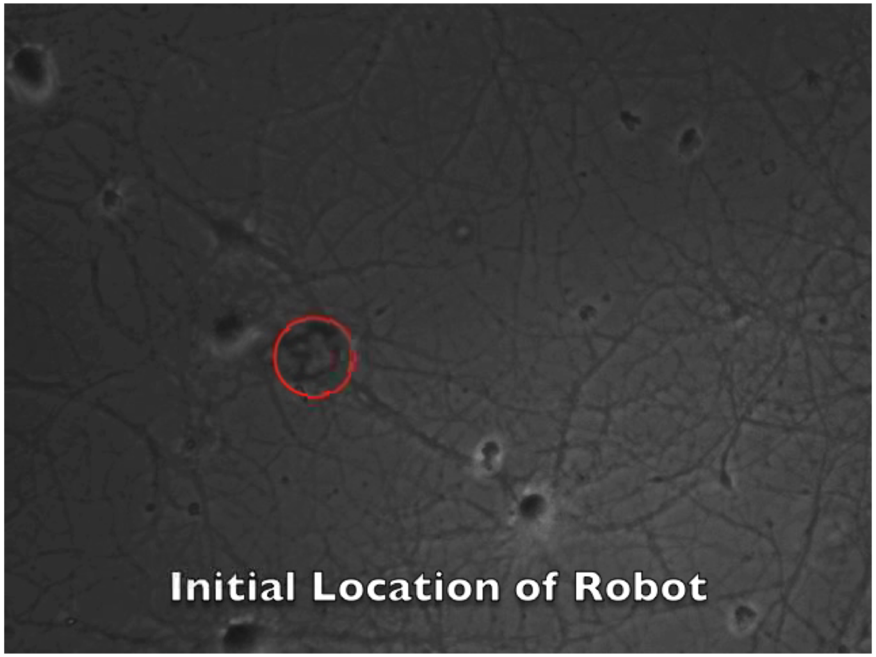


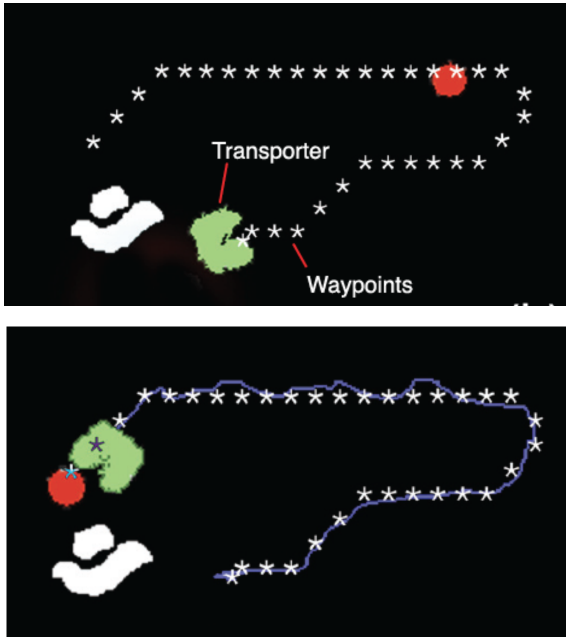
Visual feedback and autonomous transport





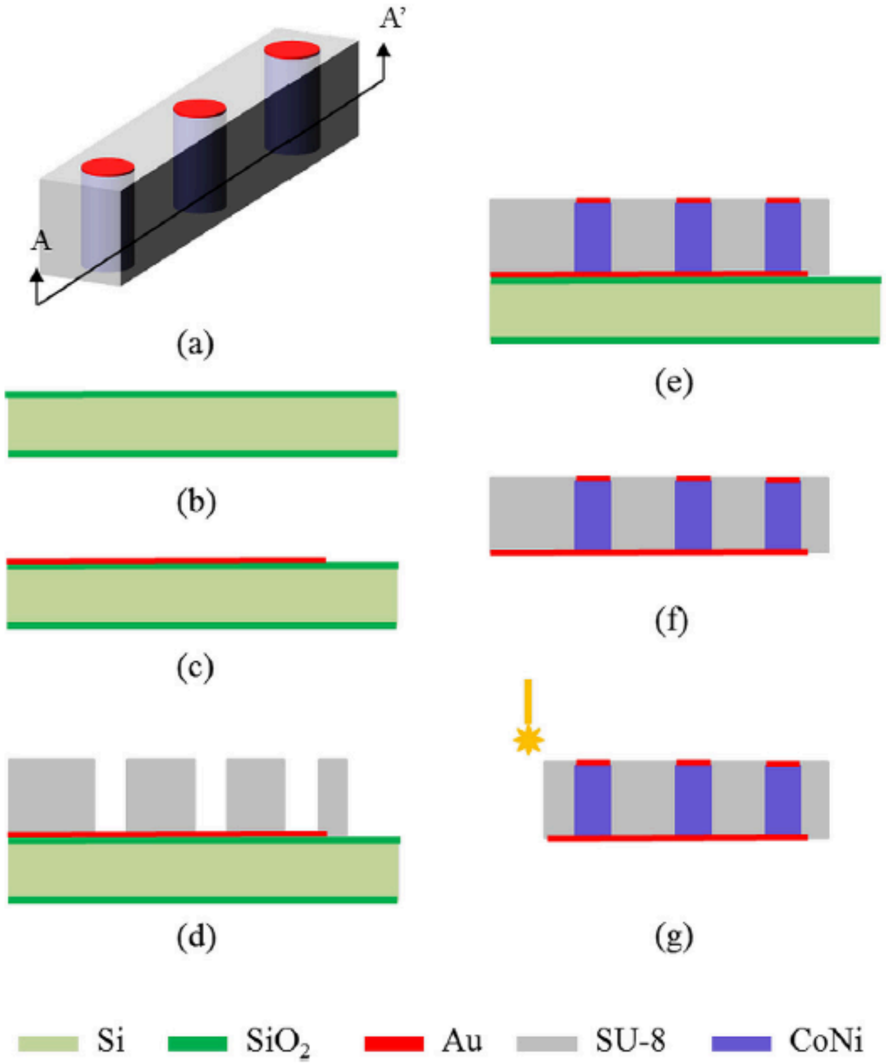
- Fluorescence labeling
- Visual tracking
- Automated manipulation
- Pick-up, transport, delivery

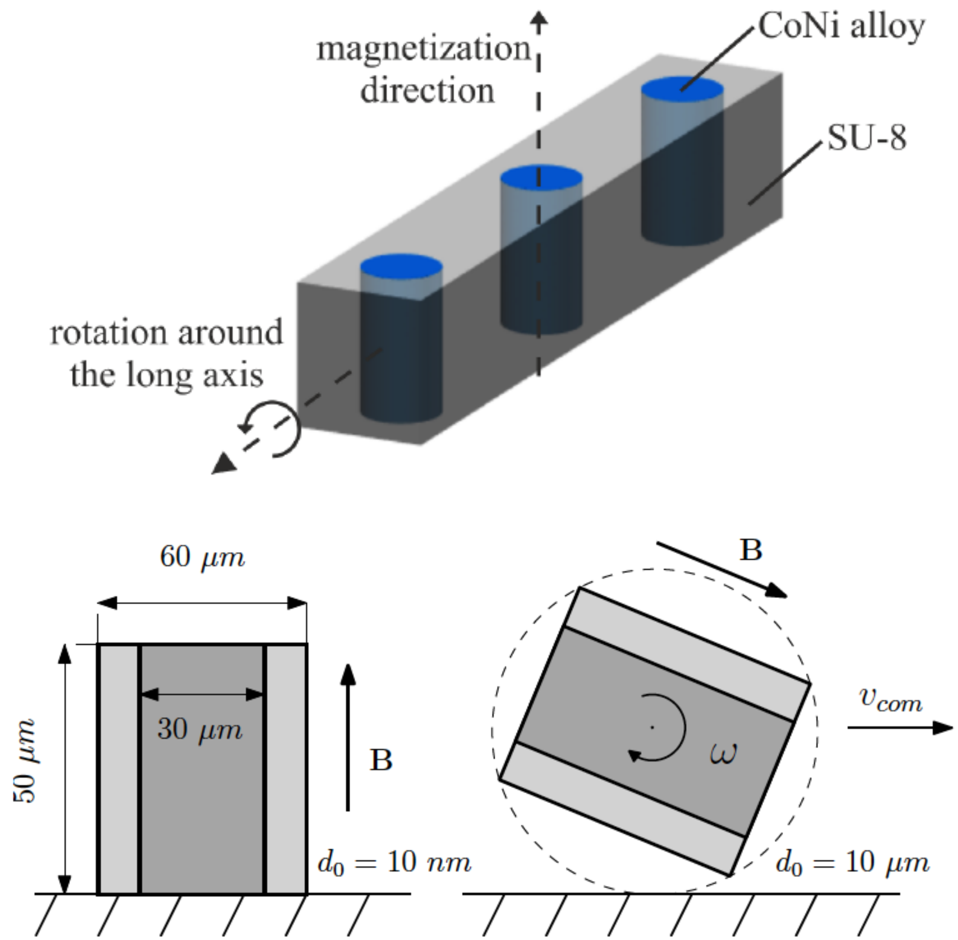




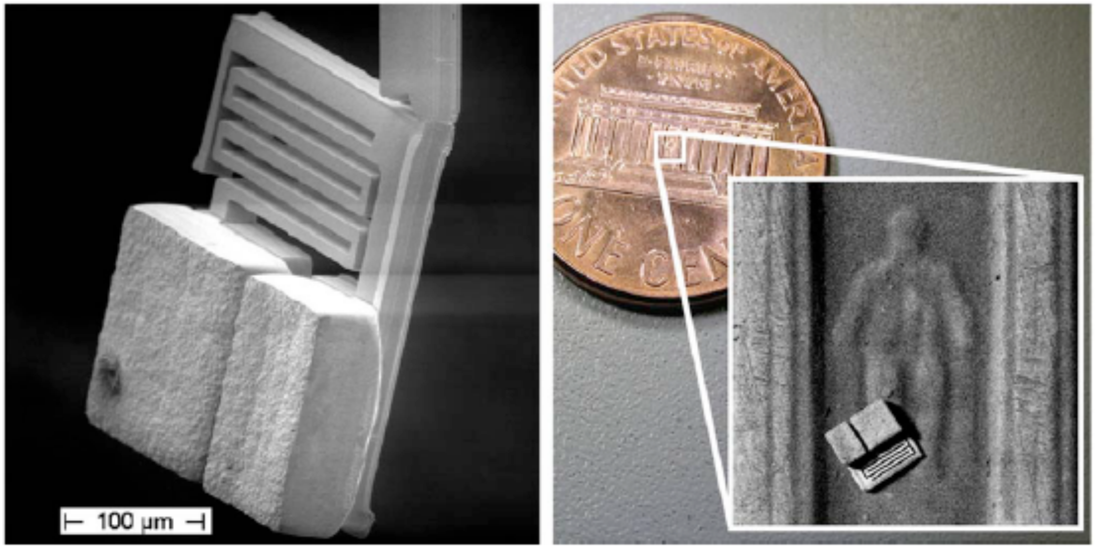
Rolling Robot

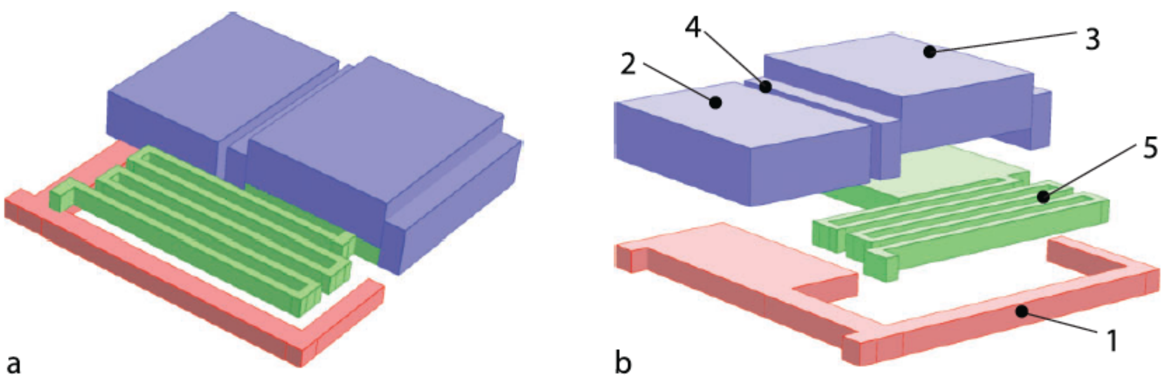
- Video





Resonant Actuators





1. Gold Base Frame
2. Nickel Attractor
3. Nickel Swinging Mass (hammer)
4. 10-20µm gap
5. Gold Spring (6µm above ground)

Vollmers etal, *APL*, 2008

Magnetic Impact Drive

- Teaser

- Power transmitted by oscillating magnetic field
  - Soft magnetic bodies in close proximity create interaction forces in the gap between them
  - Forces narrow the gap and deflect a spring separating the bodies

- Resonant Actuator: Absorb large amount of energy from the driving signal when the signal matches a natural resonant frequency

Magneto-mechanical spring-mass system